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Prírodovedecká fakulta UPJŠ
Ústav matematických vied

Fakulta elektrotechniky a informatiky TU
Katedra matematiky

11. Konferencia košických matematikov

Herľany
14. – 17. apríla 2010

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Predhovor

Milí priatelia,

vítame Vás na 11. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, Centrom excelentnosti informatických vied a znalostných systémov UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice v Herľanoch.

Cieľom konferencie je zintenzívniť stavovský život všetkých, ktorí sa v Košiciach a okolí profesionálne zaoberajú matematikou (t. j. učiteľov všetkých typov škôl, pracovníkov na poli matematických a informatických vied a aplikácií matematiky v priemysle, technike, bankovníctve a inde) a formulovať základné oblasti ich stavovských záujmov. Odborný program konferencie tradične pozostáva z pozvaných prednášok, prihlásených referátov a diskusií o stavovských problémoch. Prvé dva dni sú venované prezentácii výsledkov mladých matematikov a doktorandov. Piatkový a sobotňajší program napĺňajú prednášky pozvaných prednášateľov. V programe konferencie je vytvorený priestor aj na diskusiu o aktuálnych problémoch.

Táto štruktúra programu sa vyprofilovala z poznatkov minulých ročníkov konferencie. Doktorandom a mladším matematikom je poskytnutý priestor na získanie skúseností pri prezentácii svojich výsledkov. Je potešujúce vidieť, ako sa každým rokom zlepšujú ich vystúpenia. Veríme, že im vystúpenia na tejto konferencii pomôžu pri prezentovaní výsledkov na ďalších konferenciách. Organizačný výbor konferencie sa snaží pozývať významné osobnosti matematiky, ktoré v rámci svojich prednášok ukážu miesto matematiky v spoločenskom živote a súčasné trendy jej rozvoja. Nejedna z pozvaných prednášok mala taký pozitívny ohlas, že ich autori boli pozvaní predniesť ich aj na iných konferenciach.

Toho roku pozvanie prednášať prijali: doc. G. Andrejková (PF UPJŠ Košice), prof. P. Hanzel (UMB Banská Bystrica), prof. A. Kemnitz (TU Braunschweig, Nemecko), dr. J. Krajčiová (Gymnázium Alejová, Košice), doc. P. Maličký (UMB Banská Bystrica), prof. J. Širáň (Svf STU Bratislava) a doc. I. Žežula (PF UPJŠ Košice).

Prajeme Vám príjemný pobyt v Herľanoch

Organizačný výbor:
Stanislav Jendroľ
Ján Buša
Štefan Schrötter

16^{00} – Hanzel P. (KM PdF UMB) *Research in Mathematics Education during the Implementation of ICT in Teaching Mathematics*

17^{00} – Širáň J. (KMaDG SvF STU) *Residual Finiteness and Highly Symmetric Maps*

18^{00} – **Večera – Dinner**

19^{00} – **Spoločenský večer – Party**

Sobota – Saturday 17. 4. 2010

9^{00} – Krajčiová J. (G Alejová) *Estimating in Mathematical Education*

10^{00} – **Kávová prestávka a voľná diskusia – Coffee-break and discussion**

11^{00} – **Obed – Lunch**

Program 11. Konferencie košických matematikov

Programme of the 11th Conference of Košice Mathematicians

Streda – Wednesday 14. 4. 2010

14^{45} – **Otvorenie – Opening**

14^{50} – Hudák D. (ÚMV UPJŠ) *On Properties of Maximal 1-Planar Graphs*

15^{15} – Bezegová Ľ. (ÚMV UPJŠ) *Magic Rectangles and Multipartity Graphs*

15^{40} – Mockovčiaková M. (ÚMV UPJŠ) *Wiener Index of Graph and its Complements*

16^{05} – Ševec S. (ÚMV UPJŠ) *On the Inner Structure of the Class of Polyhedra Non-Inscribable in Spherical Shell*

16^{30} – **Kávová prestávka – Coffee-break**

17^{00} – Škrabuľáková E. (KAMA SjF TU) *Non-Repetitive List Edge-Colouring of Graphs*

17^{25} – Palenčárová D. (ÚMV UPJŠ) *Effect of the Implicit Combinatorial Model on Combinatorial Reasoning*

17^{50} – Polomčáková A. (ÚMV PF UPJŠ) *Key Competences and Discrete Mathematics*

18^{30} – **Večera – Dinner**

Štvrtok – Thursday 15. 4. 2010

9^{00} – Petrejčíková M. (ÚMV UPJŠ) *Monounary Algebras in the View of Fully Invariant Congruences and Quasiorders*

9^{25} – Šupina J. (ÚMV UPJŠ) *wQN Spaces and Semicontinuous Functions*

9^{50} – Juhás M. (ÚMV UPJŠ) *Record Values*

10^{15} – **Kávová prestávka – Coffee-break**

10^{45} – Kanáliková A. (ÚMV UPJŠ) *Irrational Numbers on the Number Line*

11^{10} – Staš M. (ÚMV UPJŠ) *Axiom of Choice and Regularity Properties on the Real Line*

their penetration into the educational process, the current situation (the role), teacher (not math) will necessarily be changed. Move to the center position of the facilitator and the examiner in the position of a coach and manager of the learning process. These changes must also respond Mathematics especially in the preparation of future mathematics teachers. In this paper we will describe the use of tools of LMS Moodle (lesson, assignment or test, evaluation) in mathematical training as well as the broader possibilities of using graphics software (analysis, arithmetic).

$[r, s, t]$-Colorings of Graphs

Arnfried Kemnitz

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Given non-negative integers r, s, and t, an $[r, s, t]$-coloring of a graph G with vertex set $V(G)$ and edge set $E(G)$ is a mapping c from $V(G) \cup E(G)$ to the color set $\{0, 1, \ldots, k-1\}$ such that $|c(v_i) - c(v_j)| \geq r$ for every two adjacent vertices v_i, v_j, $|c(e_i) - c(e_j)| \geq s$ for every two adjacent edges e_i, e_j, and $|c(v_i) - c(e_j)| \geq t$ for all pairs of incident vertices and edges, respectively. The $[r, s, t]$-chromatic number $\chi_{r,s,t}(G)$ of G is defined to be the minimum k such that G admits an $[r, s, t]$-coloring.

This is an obvious generalization of all classical graph colorings since c is a vertex coloring if $r = 1$, $s = t = 0$, an edge coloring if $s = 1$, $r = t = 0$, and a total coloring if $r = s = t = 1$, respectively.

We present general bounds for $\chi_{r,s,t}(G)$ as well as exact values for certain parameters or specific graph classes.

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Minimal k-Ranking of Antiprism

Peter Šugerek

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Given a graph G, a function $f : V(G) \to \{1, 2, \ldots, k\}$ is a *k-ranking* of G if and only if $f(u) = f(v)$ implies that every u-v-path contains a vertex w such that $f(w) > f(u)$. A *k-ranking* f is *minimal* if the reduction of any color greater than 1 violates the described ranking property. Following along the lines of the chromatic number, the *rank number of a graph* $\chi_r(G)$ is defined to be the smallest k such that G has a minimal *k-ranking*. Similarly, the concept of achromatic number can be paralleled and *arank number of a graph* $\psi_r(G)$ is defined to be a largest k such that G has a minimal *k-ranking*. We present new results involving minimal k-rankings of graphs of antiprisms.

wQN Spaces and Semicontinuous Functions

Jaroslav Šupina

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Jesenná 5, 041 54 Košice, Slovakia

The modifications for semicontinuous functions of sequence convergence properties wQN and SSP of a topological space were introduced in 2008.

point $[4\sin^2 \frac{k\pi}{2^n \pm 1}, 0]$ is a saddle point of the map F^n then there is a interior periodic point with the same itinerary with respect to the sets

$$A = \{[x, y] : 0 \le x < 2\,, 0 \le y \le 4 - x\}$$

and

$$B = \{[x, y] : 2 < x \le 4\,, 0 \le y \le 4 - x\}\,.$$

From this fact it easy to derive the existence of many interior periodic points of he map F. The aim of our talk is to show that properties of periodic points of the map F are closely related to some open problems in theory of numbers. These open poblems are *Artin's conjecture on primitive roots, Wieferich primes and Sophie Germain primes.*

Residual Finiteness and Highly Symmetric Maps

Jozef Širáň

Department of Mathematics, Slovak University of Technology,
Radlinského 11, 813 68 Bratislava, Slovakia

A group is residually finite if for any non-identity element the group contains a subgroup of finite index avoiding the chosen element. Among the important classes of residually finite groups are automorphism groups of tessellations of the infinite plane. Since any map on some surface, and in particular any 'highly symmetric' map, is a quotient of such a tessellation, residual finiteness may help proving theorems for highly symmetric maps. We outline the corresponding theory and give illustrative examples.

each housemate is assigned exactly one room, each room rent is nonnegative and the sum of all room rents is equal to the total rent for the house. Pareto optimality and envy-freeness of allocations are studied.

Key Competences and Discrete Mathematics

Anna Polomčáková

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In this paper we will bring out different views on key mathematical competences and possibilities of their development through different types of mathematical tasks. It is appropriate to focus primarily on some key competences among the varieties of topics included in school mathematics teaching content. In our contribution we focus on a thematic unit of Combinatorics and key competence Visualization, describtion of mathematical objects and situations, representation. We analyze the approach of mathematisc and IT students to solve a non-standard task of combinatorial geometry.

Axiom of Choice and Regularity Properties on the Real Line

Michal Staš

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It is well known that one cannot prove in ZF that there exists a Lebesgue non-measurable set or a set not possessing the Baire Property. For any such proof we need some assumption e.g. the Axiom of Choice AC. The Axiom of Dependent Choice AD, a weak form of AC, which is still sufficient to develop most of real analysis, is insufficient to prove (given ZF) that there is a Lebesgue non-measurable set of reals, or that there is a set of reals without the property of Baire or without the Perfect Set Property. We present a summarization of results on measure and category, mainly of regularity properties as the Lebesgue measurableness, the Baire Property and the Perfect Set Property.

Conference contributions

Magic Rectangles and Multipartity Graphs

Ľudmila Bezegová

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A magic rectangle is an $m \times n$ array the entries of which are the first mn positive integers, the rows of which have constant sum and the columns of which have constant sum. These two constants are the same just in case $m = n$ when we have magic squares. A graph is called supermagic if it admits a labelling of the edges by pairwise different consecutive positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. We consider relations between supermagic graphs and magic rectangles through new labelling of graph.

Parity Colourings of Plane Graphs

Július Czap, Stanislav Jendroľ, František Kardoš, Roman Soták, and Margit Voigt

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

We will consider vertex (edge) colourings of plane graphs. We say that a face f of a connected plane graph G uses a colour c under a colouring φ k times if this colour appears k times along the facial walk of f. Colourings of graphs embedded on surfaces with face constrains have recently drawn a substantial amount of attention. Two problems of this kind are the following:
Problem 1. A vertex colouring φ is a *weak parity vertex colouring* of a connected plane graph G if each face of G uses at least one colour an odd number of times. The problem is to determine the minimum number $\chi_w(G)$ of colours used in a such colouring of G.

Wiener Index of Graph and its Complements

Martina Mockovčiaková and Tomáš Madaras

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Jesenná 5, 041 54 Košice, Slovakia

The Wiener index $W(G)$ of a graph G is the sum of distances between all vertices of G. Entringer, Jackson, Snyder (Czech. Math. J., 1976) have studied general properties of $W(G)$ and its relation to the Wiener index of complement of G. We derive a lower bound for the sum of Wiener indices of a bipartite graph and its bipartite complement and discuss its quality; we also address the question on upper bound for this sum (both for standard and bipartite case).

Effect of the Implicit Combinatorial Model on Combinatorial Reasoning

Daša Palenčárová

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We say that the combinatorial problem is elementary if it can be solved using one combinatorial operation (permutations, combinations, variations with or without repetition). Elementary combinatorial problems may be classified into three different combinatorial models (selection, partition and distribution). In this paper, we describe the effect of the implicit combinatorial model for understanding the problem, the choice of strategies for solving and the accuracy of solving of students. We consider also the most common errors in the solutions of the problems. Based on our results we formulate ideas of the selection of combinatorial problems for mathematics lessons.

Record Values

Matej Juhás

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A sequence $\{X_n, n \geq 1\}$ of independent identically distributed random variables with common continuous distribution function $F(x)$ is considered. Random variable X_n is a record value of this sequence if $X_n > \max\{X_1, X_2, \ldots, X_{n-1}\}$. By convention X_1 is a record value. Let $\{R_n, n \geq 1\}$ be the sequence of such record values, $\{T_n, n \geq 1\}$ be the record times at which record values occur. We define $T_1 = 1$, $T_n = \min\{i; i > T_{n-1}, X_i > X_{T_{n-1}}\}$ and denote as $W_n = T_{n+1} - T_n$ the inter record times and N_n the total number of records among $X_1, \ldots, X_n$.

There are known the limit distributions of R_n, T_n, W_n and N_n. We present joint and marginal distributions of R_n for $n < \infty$ using hazard function, derive a recurrent relation for the distribution of N_n and give an algorithm for its calculation. Further we present results about a conditional distribution of W_n, T_n and give conditions for characterization of exponential records using special transformations.

Fractional Total Coloring of Complete Graphs

Gabriela Juhásová and Roman Soták

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An additive hereditary property of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphism. Let $\mathcal{P}$ and $\mathcal{Q}$ be two additive and hereditary graph properties and $r, s \in \mathbb{N}$. Then $(\mathcal{P}, \mathcal{Q})$-*fractional total coloring* of a graph G is a mapping $f : V \cup E \to \binom{\{1,2,\ldots,r\}}{s}$ such that for any color i all vertices of color i induce a subgraph from property $\mathcal{P}$, all edges of color i induce a subgraph from property $\mathcal{Q}$ and vertices and incident edges are colored differently. The minimum number of colors of a $(\mathcal{P}, \mathcal{Q})$-fractional total coloring of G is called $(\mathcal{P}, \mathcal{Q})$-*fractional*

total chromatic number $\chi''_{f,\mathcal{P},\mathcal{Q}}(G) = \frac{r}{s}$. Let $k = sup\{i : K_{i+1} \in \mathcal{P}\}$ and $l = sup\{i : K_{i+1} \in \mathcal{Q}\}$. We show for a complete graph K_n that if $l > k+1$ then $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{k+1}$ for a sufficiently large n.

Irrational Numbers on the Number Line

Andrea Kanáliková

Institute of Mathematics FSc, Pavol Jozef Šafárik University,
Jesenná 5, 041 54 Košice, Slovakia

The irrational number notation is often underrated in mathematics teaching. This article deals with Canadian research of N. Sirotic and R. Zazkis, which was aimed to understanding of irrational numbers by prospective secondary school teachers. The paper describes application of such research in Slovak condition, tests given to students of mathematics at UPJŠ and their evaluations as well as planning to further research direction.

IKT in Mathematics

Katarína Kocová Mičkaninová

Institute of Mathematics FSc, Pavol Jozef Šafárik University,
Jesenná 5, 041 54 Košice, Slovakia

Development of key competences characterized as "Key competences represent a transferable and multifunctional package of knowledge, skills and attitudes that all individuals need for personal fulfilment and development, inclusion and employment" is the main theme on streamlining the educational process. And since we live in a modern information society, ICT play an important role in the modernization of the educational process. For this reason, many countries specify standards for characterizing the using of ICT in development of key competences and ways of implementation of ICT in teaching various subjects.

BECTA (The British Educational Communications and Technology Agency) created in the frame of the Key Stage 3 National Strategy the model developing key ICT competence in which is defined the most relevant Key ICT competences for teaching mathematics. In this article we will talk about this model and its applications in teaching mathematics.

How Pupils Assess Probabilities of Some Events

Mária Kolková

Institute of Mathematics FSc, Pavol Jozef Šafárik University,
Jesenná 5, 041 54 Košice, Slovakia

This report is based on our research in which we investigate probabilistic thinking of pupils of the 8th and 9th class of an elementary school. As a method to assess probability we used colouring of a part of a rectangle. This special method enabled us to judge whether involved pupils computed or only estimated probability of a given event. Our findings will be demonstrated via chosen analysed tasks.

On the Crossing Numbers of Cartesian Products of Small Power Graphs

Daniela Kravecová

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The *crossing number*, $cr(G)$, of a graph G is the minimal number of pairwise intersections of nonadjacent edges in any drawing of G in the plane. Computing the crossing number of a given graph is in general an elusive problem. The exact values of the crossing numbers are known only for some families of graphs. Patil and Krishnnamurthy established family of graphs for which power graphs have crossing number one. There are known several exact results of the crossing numbers of the Cartesian product of a small graphs with paths, cycles and stars.

In the talk we give several exact values of the crossing numbers for the Cartesian products of some small power graphs and path P_n.

Problem 2. A vertex colouring φ is a *strong parity vertex colouring* of a 2-connected plane graph G if for each face f and each colour c, no vertex or an odd number of vertices incident with f are coloured by c. The problem is to find the the minimum number $\chi_s(G)$ of colours used in a such colouring of G.

Similarly we can define weak and/or strong parity edge colouring.

We will give a survey on results and open problems concerning the above problems.

On Properties of Maximal 1-Planar Graphs

Dávid Hudák

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A graph is called *1-planar* if it can be drawn into the plane so that each its edge is crossed by at most one another edge. A graph G from the family $\mathcal{G}$ of graphs is *maximal* if $G + uv \notin \mathcal{G}$ for any two nonadjacent vertices $u, v \in V(G)$. We deal with the properties of maximal 1-planar graphs compared with maximal planar graphs. We study these graphs from the view of properties of the number of edges, different diagrams, local structure and hamiltonicity.

Monounary Algebras in the View of Fully Invariant Congruences and Quasiorders

Danica Jakubíková–Studenovská and Mária Petrejčíková

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A congruence (quasiorder) of an algebra (A, f) is called fully invariant (fully r-invariant), if the congruence (quasiorder) respects all endomorphisms (retractions) of the algebra. We characterize all monounary algebras (A, f) having one of the following properties : has only fully invariant congruences (CFI), has only fully invariant quasiorders (QFI), has only fully r-invariant congruences (CRI) or has only fully r-invariant quasiorders (QRI).

Cartesian Products with Small Crossing Number

Jana Petrillová

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Computing the exact value of crossing number of a given graph is in general an elusive problem. Let G_1 and G_2 be simple graphs with vertex sets $V(G_1)$ and $V(G_2)$, and edge sets $E(G_1)$ and $E(G_2)$. The Cartesian product $G_1 \times G_2$ of graphs G_1 and G_2 has vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and any two vertices (u, u') and (v, v') are adjacent in $G_1 \times G_2$ if and only if either $u = v$ and u' is adjacent with v' in G_2, or $u' = v'$ and u is adjacent with v in G_1. There are several known exact results on the crossing numbers of Cartesian products of paths and cycles with other graphs. The aim of this presentation is to characterize graphs G_1 and G_2 for which the crossing number of their Cartesian product is small. Tindira in his MSc Thesis proved the necessary and sufficient conditions for all pairs of graphs G_1 and G_2 for which the crossing number $G_1 \times G_2$ is one. We extend this result and proved the necessary and sufficient conditions for the case when the crossing number $G_1 \times G_2$ equals two. This problem is divided into two parts. First we solve the case if one of the graphs G_1 and G_2 is a cycle and then the case when one of the graphs G_1 and G_2 is a path.

The Housemates Problem

Eva Pillárová

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We are concerned with a housemates problem, a variant of the fair division problem proposed by Brams and Kilgour (2001): there are n rooms in a house that n housemates rent, a positive total rent for the house and a nonnegative bid from each housemate for each room. We want to allocate the rooms to the housemates and determine room rents in such a way that

Logistic, Multinomial, and Ordinal Regression

Ivan Žežula

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While linear regression model has its firm place in basic or intermediate statistics courses and textbooks, its generalizations have much worse position. Even if nominal and ordinal data are frequently met in many applications, the sweetness of normality assumption seems to override everything. We will try to explain the basic principles of regression methods for categorical data, looking at both mathematical foundations and practical aspects.

On the Inner Structure of the Class of Polyhedra Non-Inscribable in Spherical Shell

Sergej Ševec

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Types of polyhedra are studied that are non-inscribable in spherical shell. A distinction is raised due to the trimerous scale of polyhedra non-inscribability kinds: non-inscribability in sphere, normal non-inscribability, and spherical-shell non-inscribability. There is a duality aspect involved in the definition of spherical-shell non-inscribability that naturally also brings in definitions dual to the two other kinds of non-inscribability. Adequate refined set of non-inscribability kinds is produced. Based on it, some graph-combinatorial conditions and constructions are formulated to manifest infiniteness of the varieties of types discriminating the refined kinds of polyhedra non-inscribability. As a result, the fundamental classification system for the types of polyhedra non-inscribable in spherical-shell is accomplished.

Non-Repetitive List Edge-Colouring of Graphs

Erika Škrabuľáková and Jens Schreyer

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Let G be a simple graph and let φ be a colouring of its edges by symbols from the set A, $r_1, r_2, r_3, \ldots, r_{2n} \in A$, $n \geq 1$. A sequence $r_1, r_2, \ldots, r_{2n}$ with $r_i = r_n + i$ for all $i = 1, 2, 3, \ldots, n$ is called a repetition. A sequence S is called non-repetitive if none of its blocks forms a repetition. An edge-colouring φ is non-repetitive if for any simple path in G the associated sequence of colors is non-repetitive. Moreover if the colour of every edge e is chosen only from pre-assigned list of colours L_e we speak about non-repetitive list edge-colouring of the graph G.

We show several ideas how to find a non-repetitive edge-colouring of a graph if the colours of its edges are chosen only from lists associated to these edges.

Estimating in Mathematical Education

Jana Krajčiová

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The contribution describes the reasons to introduction of estimates to mathematical education. Fields of mathematics working with estimates are listed. Different types of estimates used in the educational process are showed. The experiments which based on phenomena breaking the usual intuition (e. g. the Monty Hall paradox) are described.

Dynamics of a Map of a Triangle and Some Open Problems in Theory of Numbers

Peter Maličký

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Given the plane triangle

$$D = \{\, [x, y] : 0 \le x,\, 0 \le y,\, x + y \le 4 \,\}$$

we consider the map

$$F : D \to D,\ [x, y] \mapsto [x(4 - x - y),\, xy]$$

and its periodic points. It is easy to show that a point $P = [x, 0] \in D$ is a periodic point of the map F if and only if $x = 4\sin^2 \frac{k\pi}{2^n \pm 1}$, where n and k are positive integers with $2k < 2^n \pm 1$. We are interested in *interior* periodic points of the map F. It is easy to verify that the point $[1, 2]$ is a fixed point of the map F. In 2004 it was found the point $\left[1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right]$ with period 4. We have discovered the point $\left[1, \frac{3+\sqrt{5}}{2}\right]$ with period 6. Our main result is a relation between lower and interior periodic points. Namely, if a

Continuous functions were substituted by lower and upper semicontinuous ones in their definitions. We present the relations among the original properties and these new modifications, we discuss some special intervals as the ranges of the functions in the definitions of the properties and we give the relations among such modified notions.

On Max-Min Problem for Particular Graphs Family – Hereditary Properties

Martin Tajboš

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For a graph $G = (V, E)$ and $e = \{x, y\} \in E(G)$ the weight of e is defined as $w(e) = deg(x) + deg(y)$. Erdős asked the question what is the minimum weight of an edge e of a graph G having n vertices and m edges? Let $\mathcal{G}(n, m)$ be the family of all graphs having n vertices and m edges. Motivated by Erdős's question Ivančo, Jendroľ [1] solved the problem

$$W(n, m) = \max_{G \in \mathcal{G}(n,m)} \left\{ \min_{e \in E(G)} w(e) \right\}$$

Let $\mathcal{P}$ be a given property of graphs and $\mathcal{G}(n, m, \mathcal{P})$ be the family of all graphs having n vertices, m edges and property $\mathcal{P}$. Then

$$W(n, m, \mathcal{P}) = \max_{G \in \mathcal{G}(n,m,\mathcal{P})} \left\{ \min_{e \in E(G)} w(e) \right\}.$$

We study the behavior of $W(n, m, \mathcal{P})$ with respect to different hereditary properties of graphs, i.e. properties that are closed with respect to taking subgraphs.

References

[1] J. Ivančo, S. Jendroľ, On extremal problems concerning weights of edges of graphs, Colloquia Mathematica Societatis János Bolyai, **60** (1991), 399–410.

Invited lectures

Models of Artificial Neural Networks, their Capability and Applications

Gabriela Andrejková

Institute of Computer Science FSc, Pavol Jozef Šafárik University, Jesenná Street 5, 040 01 Košice, Slovakia

The contribution is prepared as an overview of some neural networks models and their properties in the following contenst: Biological motivations to build artificial models of neural networks. Linear and polynomial thereshold gates and their capabilities. Feed-forward and recurrent neural networks, back propagation algorithm to adaptation of neural networks, a capability of neural networks to be universal approximators. Using of recurrent neural networks in some applications (prediction of geomagnetic storms). Hopfield neural networks and their applications (longest common subsequence of two strings). Neural networks in connections to computational models.

Research in Mathematics Education during the Implementation of ICT in Teaching Mathematics

Hanzel Pavol

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Dynamic development of educational management systems (LMS) and the growing availability of intelligent mathematical software, significantly affect the modalities and teaching methods in mathematics. In parallel with the rapid development of information and communication technologies and

11^{35} – Pillárová E. (ÚMV UPJŠ) *The Housemates Problem*

12^{00} – Kocová Mičkaninová K. (ÚMV PF UPJŠ) *IKT in Mathematics*

12^{30} – **Obed – Lunch**

14^{50} – Tajboš M. (ÚMV ÚPJŠ) *On Max-Min Problem for Particular Graphs Family – Hereditary Properties*

15^{15} – Kravecová D. (KM FEI TU) *On the Crossing Numbers of Cartesian Products of Small Power Graphs*

15^{40} – Petrillová J. (KM FEI TU) *Cartesian Products with Small Crossing Number*

16^{05} – Juhásová G. (ÚMV UPJŠ) *Fractional Total Coloring of Complete Graphs*

16^{30} – **Kávová prestávka – Coffee-break**

17^{00} – Czap J. (ÚMV UPJŠ) *Parity Colourings of Plane Graphs*

17^{25} – Šugerek P. (ÚMV UPJŠ) *Minimal k-Ranking of Antiprism*

17^{50} – Kolková M. (ÚMV PF UPJŠ) *How Pupils Assess Probabilities of Some Events*

18^{30} – **Večera – Dinner**

Piatok – Friday 16. 4. 2010

8^{30} – Maličký P. (KM FPV UMB) *Dynamics of a Map of a Triangle and Some Open Problems in Theory of Numbers*

9^{30} – **Kávová prestávka – Coffee-break**

10^{00} – Žežula I. (ÚMV UPJŠ) *Logistic, Multinomial, and Ordinal Regression*

11^{00} – Kemnitz A. (TU Braunschweig, Germany) *$[r, s, t]$-Colorings of Graphs*

12^{30} – **Obed – Lunch**

14^{30} – Andrejková G. (ÚIF UPJŠ) *Models of Artificial Neural Networks, their Capability and Applications*

15^{30} – **Kávová prestávka – Coffee-break**

Obsah – Contents

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