Jednota slovenských matematikov a fyzikov Pobočka Košice

> Prírodovedecká fakulta UPJŠ Ústav matematických vied

Fakulta elektrotechniky a informatiky TU Katedra matematiky a teoretickej informatiky

# 19. Konferencia košických matematikov

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#### Predhovor

Vážení priatelia, milí hostia, kolegyne a kolegovia,

vitajte na 19. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice – v Herlanoch.

Nápad organizovať konferenciu tohto typu vznikol v našej pobočke JSMF pod vedením prof. Jendroľa pred viac ako osemnástimi rokmi. Bola za tým myšlienka, že ľudia profesionálne sa zaoberajúci matematikou v jej rôznych podobách (učitelia, vedci, aplikovaní matematici) a žijúci na východe Slovenska by mali mať možnosť sa pravidelnejšie stretávať, podeliť sa s rovnako "postihnutými" kolegami o svoje radosti i starosti súvisiace s prácou matematika či matematikára; následne spoločne alebo s ďalšími spriaznenými dušami hľadať riešenia či východiská z problémov. Prípadne si vzájomne pomáhať a povzbudiť sa navzájom. Ďalej to bola predstava, že by malo ísť o serióznu konferenciu s kvalitným obsahom, najmä pozvanými prednáškami. Od začiatku boli na ňu pozývaní prednášajúci s cieľom, aby to boli či už zrelé alebo práve vychádzajúce kvalitné osobnosti, známe vo svojom prostredí, s cieľom dozvedieť sa nové veci, nadviazať nové či upevniť staré kontakty. Viaceré z týchto prednášok mali taký pozitívny ohlas, že ich autori boli pozvaní prednášať aj na iných konferenciách.

To, že Konferencia košických matematikov sa koná po 19. krát je len potvrdením, že tieto myšlienky našli úrodnú pôdu. Každoročne sme na nej mali skvelých prednášajúcich. Na výbere a príprave konferencie sa pracuje celý rok. O výbere pozvaných prednášajúcich sa v podstate rozhoduje na tradičnom každoročnom stretnutí výboru košickej pobočky JSMF s košickými profesormi matematiky a vedúcimi košických matematických pracovísk, vrátane riaditeľa Gymnázia na Poštovej ulici, ktoré má matematické triedy.

Zostáva minuloročná nová štruktúra konferencie. Prvý deň (štvrtok) je venovaný najmä mladým začínajúcim matematikom. Mnohí dnes už veľmi úspešní kolegovia mali svoje prvé verejné odborné či vedecké vystúpenie práve na našej konferencii. Vystúpenia mladých kolegov majú z roka na rok vyššiu úroveň, čo organizátorov veľmi teší. V piatok a v sobotu dopoludnia sa konajú najmä pozvané prednášky, aby sa na nich mohlo zúčastniť čo najviac účastníkov. Spoločenský piatkový večer je organizovaný tak, aby bolo možné v menších skupinách pri poháriku vínka predebatovať rôzne otázky. Aj tento rok sa nám podarilo získať viacero výrazných osobností. Pozvanie prednášať prijali: doc. RNDr. Vladimír Baláž, CSc. (OM ÚIAM FCHPT STU Bratislava), prof. Dr. Vasile Berinde (DM&CS, NUC of TU Cluj Napoca, Baia Mare, Rumunsko), doc. RNDr. Jiří Demel, CSc. (KII SvF ČVUT v Prahe), RNDr. Monika Dillingerová, PhD. (KAGaDM, FMFI, UK Bratislava), prof. RNDr. Stanislav Jendroľ, DrSc. (ÚMV PF UPJŠ Košice), dr. Borut Lužar (FIS in Novo mesto, Slovenia), doc. RNDr. František Staněk, Ph.D. (KMaDG VŠB Ostrava), doc. RNDr. János Tóth, PhD. (KMaI EkF ÚJS Komárno).

Tento rok je rokom významného jubilea dlhoročného organizátora našej konferencie profesora Stanislava Jendroľa. Pri tejto príležitosti mu do nasledujúcich rokov prajeme všetko najlepšie, najmä zdravie, rodinnú pohodu ale aj ďalšie tvorivé úspechy.

Prajeme vám príjemný pobyt v Herľanoch

Organizačný výbor:

Ján Buša Jozef Doboš Róbert Hajduk

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## Invited lectures

## On Generalized Notion of Convergence by Means of Ideal and Its Applications Vladimír Baláž

Faculty of Chemical and Food Technology, Slovak University of Technology in Bratislava, Radlinského 9,812 37 Bratislava, Slovak Republic

The starting point of this talk is the notion of  $\mathcal{I}$ -convergence, in this way introduced by P. Kostyrko, T. Šalát and W. Wilczyński by means of ideal  $\mathcal{I}$  [3], (see also [1] where  $\mathcal{I}$ -convergence is defined by means of filter – the dual notion to ideal).  $\mathcal{I}$ -convergence is the natural generalization of the notion of statistical convergence. The notion of statistical convergence was independently introduced by H. Fast (1951) [2] and I. J. Schoenberg (1959) [3] which generalized the notion of classical convergence and has been developed in several directions and used in various parts of mathematics, in particular in number theory, mathematical analysis and ergodic theory. This talk would like to point out the usefulness of  $\mathcal{I}$ -convergence mainly in number theory.

- Burbaki, N.: Éléments de Mathématique, Topologie Générale Livre III. (Russian translation: Obščaja topologija Osnovnye struktury.) Nauka, Moscow 1968.
- [2] Fast, H.: Sur la convergence statistique. Colloquium Mathematicae, No-2, 1951, 241–244.
- [3] Kostyrko, P., Šalát, T., Wilczyński, W.: *I-Convergence*. Real Anal. Exchange, No-26, 2000, 669–686.
- [4] Schoenberg, I. J.: The Integrability of Certain Functions and Related Summability Methods. The American Mathematical Monthly, No-66, 1959, 361–375.

## Pompeiu-Hausdorff Metric and Its Wide Spreading Role in Science and Technology

#### Vasile Berinde

Department of Mathematics and Computer Science, North University Centre at Baia Mare, Technical University of Cluj-Napoca, Romania

The Romanian mathematician Dimitrie Pompeiu (1873–1954) defined the distance between two curves in the complex plane in his PhD Thesis defended in 1905 at Sorbonne University Paris and published in the same year in [D. Pompeiu, *Sur la continuité des fonctions de variables complexes* (Thèse). Gauthier-Villars, Paris, 1905; Ann. Fac. Sci. de Toulouse **7** (1905), 264–315].

Almost ten years later, the famous German mathematician Felix Hausdorff (1878–1942) reconsidered in his book [F. Hausdorff, *Grundzuege der Mengenlehre*. Veit & comp., Leipzig, 1914; 473 Pag.] this concept in the more general framework of a metric space, a concept introduced in the meantime by the famous French mathematician Maurice Fréchet (1878–1973) in [M. Fréchet, *Sur quelques points du calcul fonctionnel* (Thése), Rend. Circ. Mat. Palermo. **22** (1906), 1–74].

The importance of this fundamental concept came rather late in mathematics (about 1940) and even later in applied sciences but nowadays it is widely used in almost all research areas. The list of applications of what is generally known as Hausdorff metric and less often as Pompeiu-Hausdorff metric is really impressive.

The main aim of this talk is to give a brief account on Pompeiu-Hausdorff metric and its ubiquity in applied sciences and technology.

## Generating Exercises for Tutorials and Assessment

#### Jiří Demel

Department of Applied Informatics, Faculty of Civil Engineering, Czech Technical University in Prague, Thákurova 7, 166 29 Praha 6, Czech Republic

When preparing written part of an assessment one has to get enough "reasonable" exercises to be solved by students.

What we mean by "reasonable"? An instance of a problem should not be too easy and too difficult because of the time limit for a written examination. The process of solution should require knowledge of a method or of some important tricks. Instances with trivial solutions should be avoided. Unknowingness of the method or of the tricks should be detectable through a wrong result.

The result and in some cases also important steps of the solution should be easily verifiable. For example, an optimization problem may be required to have a single optimal solution. Last but not least, the instance of the problem should not be publicly accessible e.g. on the Internet. Ideally, each student should be given a unique instance of the problem which will never be used again.

Unfortunately, obtaining instances of problems as mentioned above manulally is usually a tedious task. Here a computer can help.

The straightforward strategy is to randomly generate an instance of the problem, solve it, evaluate its suitability, and if refused generate another one. For some problems the evaluation of suitability is not easy.

The talk will discuss some nontrivial methods of generating suitable exercises from the field of graph theory and operations research.

## Logic in the Admissions Interviews at Universities

#### Monika Dillingerová

Faculty of Mathematics, Physics and Informatics, Comenius University, Mlynská dolina, 842 48 Bratislava

There have been many changes in teaching mathematics over the last 20 years. This is accompanied by a subsequent change in the form and content of admissions interviews at universities. One of the dynamically changing areas is logic. In the "classical" interviews we still find a formal sentential logic. Nowadays, in the tests produced by different companies, it is mostly the use of logic in various life situations, fictional worlds, and the correct understanding of quantified statements. The article highlights the types of tasks assigned to tests by companies like Scio and others. It offers their distribution according to the focus. It also shows how to solve such tasks with pupils. All the proposed tasks and exercises were tested over three years on students prepared for admissions at the Comenius University Center for Continuing Education in Bratislava.

#### Mathematics, Chemistry and Nobel Prizes

#### Stanislav Jendroľ

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

In the talk two special convex polyhedra will be considered and their role as historical milestones in combinatorial mathematics, computer science and theoretical chemistry will be illustrated. The main part of the talk will focus on Hamiltonian cycles in convex polyhedra and in fullerenes.

## Colorful Graph Theory

#### Borut Lužar

Faculty of Information Studies, Novo mesto, Slovenia & Faculty of Science, Pavol Jozef Šafárik University, Košice, Slovakia

Graph theory has gained an unexpected momentum about a century after Euler set the foundations of the field by solving the Bridges of Königsberg Problem. Its cause has been a conjecture, known as the Four Color Problem, for which another century needed to pass in order to be answered in affirmative. The conjecture attracted scientists from all fields of mathematics and contributed to development of new methods not only in discrete mathematics, but also in algebra, topology and others. Chromatic graph theory is at present still one of the most prominent branches of graph theory, divided in a number of research directions.

The fundamental problem in chromatic graph theory is to specify the least number of colors with which the vertices of a graph can be colored in such a way that the neighboring vertices receive distinct colors. Of course, restricting graphs to particular classes usually enables us to set different, better bounds for the minimum number of colors as additional properties are guaranteed. Line graphs are representatives of such a particular class. Clearly, coloring the vertices of the line graph of a graph G is equivalent to coloring the edges of G, and for this class Vizing proved that the required number of colors is equal either to the maximum degree of a graph or one color more. This was one of the facts enabling the edge-coloring to become one of the most flourishing fields of chromatic graph theory.

It was therefore expected that other chromatic invariants having vertices for their domains obtained their variations in the domain of edges also. In our talk, we will introduce the main problems which are currently 'hot topics' in this field of research; namely, the Strong Edge-coloring Conjecture [1], the 1-2-3 Conjecture [2], and the Perfect 1-Factorization Conjecture [3]. We will finally present some recent results on these topics and conclude with possible further research directions.

Acknowledgement. The author acknowledges support by the Slovenian Research Agency Program P1–0383.

- P. Erdős. Problems and results in combinatorial analysis and graph theory. In Proceedings of the First Japan Conference on Graph Theory and Applications, pages 81–92, 1988.
- [2] M. Karoński, T. Łuczak, and A. Thomason. Edge weights and vertex colours. J. Combin. Theor. Ser. B, 91(1):151–157, 2004.
- [3] A. Kotzig. Hamilton graphs and Hamilton circuits. In Theory of graphs and its applications, Proceedings of the Symposium of Smolenice 1963, Nakl SAV, Praha, pages 62–82, 1964.

## Methodology and Algorithms for Creating and Visualization of 3D Models for Mineral Deposits of Critical EU Commodities

#### František Staněk and Jana Staňková

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This contribution focuses on research within the project TE02000029 Competence Centre for Effective and Ecological Mining of Mineral Resources, granted by the Technology Agency of the Czech Republic. The main goal of this project is revision of the deposits of selected non-energetic raw materials, which belong to critical EU commodities [1]. Its Work Package WP4 "Spatial modeling of mineral deposits" focuses on digital modeling of selected deposits, using appropriate mathematical techniques, based on the study and evaluation of the archived data. Within this Work Package 3D models of deposits were made: Li-Sn-W deposit Cinovec east [2], kaolin deposit Jimlíkov – east (located in the neighborhood of the village Jimlíkov, about 5 km west of Karlovy Vary) [3, 4] and graphite deposit Český Krumlov – Městský vrch [5].

In this contribution we describe individual steps for creation and visualization of 3D models of the deposit, including all steps from reevaluation of all accessible archived materials and verification and correction of the input data, up to the visualization of categories of the blocks of reserves. Using a specialized software we check all input data, compatibility of used software, and generate outputs: estimates of the deposit reserves in a textual form and various types of visualization of the deposit in 2D and 3D, respectively. This methodology and a newly developed software allow us to both create variant models of the kaolin deposits of this and a similar types and fast updates of the models when the input data and/or the parameters of the model are updated and/or amended. Our dynamic complex model for kaolin deposits can be updated when needed, based on mining explorations including variant estimates of the resources based on a priori defined utility conditions.

In our modeling standard software tools, such as MS Excel, Surfer [6] and Voxler [7] made by the Golden Software Company, and an open-source software SGeMS [8], are used. For creating macros in the MS Office environment we used the Visual Basic for Applications (VBA), while for individual software tools we used the standard Visual Basic.

Acknowledgement. This work was supported by the project TAČR TE02000029 – CEEMIR (Competence Centre for Effective and Ecological Mining of Mineral Resources).

- Competence Centre for Effective and Ecological Mining of Mineral Resources. Internet: https://www.hgf.vsb.cz/ceemir/en/, [Feb. 28, 2018].
- [2] Staněk, F., Jelínek, J., Grygar, R., Jarošová, M.: Metodika tvorby a vizualizace 3D modelu Li-Sn-W ložiska Cínovec - východ. VŠB-TU Ostrava, 2017. Internet: https://www.hgf.vsb.cz/ceemir/cs/vystupy/ Vysledky-a-zpravy/, [Feb. 28, 2018].
- [3] Staněk, F., Jelínek, J., Grygar, R., Jarošová, M.: Metodika tvorby a vizualizace 3D modelu ložiska kaolinu Jimlíkov – východ. VŠB-TU Ostrava, 2017. Internet: https://www.hgf.vsb.cz/ceemir/cs/vystupy/ Vysledky-a-zpravy/, [Feb. 28, 2018].
- [4] Staněk, F., Jarošová, M., Staňková, J.: Dynamic model of a kaolin deposit. Management Systems in Production Engineering. 2017, 6, 4, 179– 190.
- [5] Staněk, F., Jelínek, J., Grygar, R., Franěk, J., Jarošová, M.: Metodika tvorby a vizualizace 3D modelu ložiska grafitu Český Krumlov – Městský vrch. VŠB-TU Ostrava, 2017. Internet: https://www.hgf.vsb.cz/ ceemir/cs/vystupy/Vysledky-a-zpravy/, [Feb. 28, 2018].
- [6] SURFER Explore the Depths of your data. Internet: http://www.goldensoftware.com/products/surfer/features, [Feb. 28, 2018].
- [7] VOXLER Power Forward into 3D Visualization. Internet: http: //www.goldensoftware.com/products/voxler/features, [Feb. 28, 2018].
- [8] Remy, N., Boucher, A., Wu, J.: Applied Geostatistics with SGeMS: A User's Guide. New York: Cambridge University Press, 2009, xix, 264 pp.

## On Partial Limits of Sequences János T. Tóth

Department of Mathematics and Informatics, Faculty of Economics, J. Selye University, Bratislavská cesta 3322, 945 01, Komárno, Slovakia

This talk is based on a common paper with L. Mišík.

The concept of a limit of a sequence is a basic concept in mathematical analysis. We analyse this concept in more details using another basic concept of analysis, the concept of measure on sets of positive integers. We de

ne a degree of convergence of a given sequence to a given point with respect to a chosen measure as a number in interval [0; 1]. By a measure on  $\mathbb{N}$  we will mean any monotone function  $\mu : 2^{\mathbb{N}} \to [0; 1]$  such that  $\mu(\emptyset) = 0$ and  $\mu(\mathbb{N}) = 1$ .

**Definition.** Let (X; d) be a metric space,  $\boldsymbol{x} = (x_n)$  a sequence in X and let  $x_0 \in X$ . Suppose that  $\mu$  is a measure on N. The number

$$L_{\mu}(\boldsymbol{x}; x_0) = \inf \{ \mu(\{n \in \mathbb{N} | d(x_n; x_0) < \varepsilon\}); \varepsilon > 0 \}$$

is called the **degree of convergence** of the sequence  $\boldsymbol{x}$  to the point  $x_0$  with respect to the measure  $\mu$  and we say that  $\boldsymbol{x}$  converges to  $x_0$  to degree  $L_{\mu}(\boldsymbol{x}; x_0)$ . We call a point  $x_0 \in X$  a  $\mu$ -limit of a sequence  $\boldsymbol{x}$  if  $L_{\mu}(\boldsymbol{x}; x_0) = 1$ and we will denote this fact by  $\boldsymbol{x} \xrightarrow{\mu} x_0$ . In this case we will also say that  $\boldsymbol{x}$ is  $\mu$ -convergent to  $x_0$ .

We study properties of partial convergence depending on properties of the chosen measure. It appears that standard limits and their known generalizations (convergence with respect to a

lter or ideal) are special cases in our approach. For special kinds of measures we prove generalizations of theorems of uniqueness of limits, convergence of subsequences and preservation of arithmetic operations. In all cases the standard theorems simply follow from our theorems.

We present several examples including those showing that our theory includes the most known generalizations of the concept of limit, i.e. various kinds of the  $\mathcal{I}$ -convergence, or equivalently, convergence with respect to special filters.

## **Conference contributions**

## RULES MATH: New Rules for Assessing Mathematical Competencies

#### Marie Demlova

Department of Mathematics, FEE CTU in Prague, Prague, Czech Republic

During the last decades, different ways were introduced into mathematics teaching, especially into mathematics teaching of future engineers. For example, let us mention project oriented teaching, blended teaching, student oriented learning, and others. In 2002 Morgens Niss with his colleagues brought into consideration eight principal competencies that should be gained during mathematics education – see "Mathematical competencies and the learning of mathematics: The Danish KOM project".

On the other hand, every university teacher very quickly discovers that students learn mainly what will be assessed. Therefore, if we want students to gain some competencies we should incorporate them into our assessing process.

With this in mind, in 2017 a group of mathematics teachers from nine universities in eight European countries (with the project leader University of Salamanca, Spain) proposed an Erasmus+ project with the title *RULES MATH: New Rules for assessing Mathematical Competencies.* The project has the following two main goals:

- 1. To help developing a collaborative, comprehensive and accessible competence-oriented assessment model/models for mathematics in engineering content.
- 2. To elaborate and collect resources and materials suitable for competence-oriented assessment.

The talk will present the project in more details.

Acknowledgement. Financial support of the Erasmus+ project 2017-1-ES01-KA203-038491 "New Rules for assessing Mathematical Competencies" is gratefully acknowledged.

## A View into the Workbooks and Textbooks of Mathematics

#### Jozef Doboš

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

**Motto**: One of the tasks of a teacher of Mathematics is to make the materials as easy as possible "while still being correct".

In this submission we look at the primary school workbooks and textbooks of Mathematics from this particular angle.

Acknowledgement. The research has been supported by VEGA grant 1/0265/17.

## On an Optimal Distributions of Issues and Costs Reducing via Graph Algorithms

Elena Drabiková and Erika Fecková Škrabuľáková

Institute of Control and Informatization of Production Processes, Faculty BERG, Technical University of Košice, Němcovej 3, 042 00 Košice, Slovakia

Graph algorithms are very useful for reducing costs in different areas of business activities. As an illustration of real implementation of graph models and algorithms in the cost saving task we study a problem of optimal distribution of information desks in a large exhibition hall. This problem is similar to a problem of finding k clusters (attraction zones of each information desk) that minimize the maximum diameter of each cluster. The difference between this task and a task solved in our contribution is in its discrete structure (the move of a visitor is allowed only along paths in a graph) and in the fact that in our case the position of two information desks is predefined. The more optimal is the position of information desks detected, the lower are the organizational costs.

We also deal with an identification of optimal allocation of guide signs via graph theory models. Although our implementation model concerns distribution of issues in a large exhibition hall, similar algorithm could be utilized in many other navigational situations.

Acknowledgement: This work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0892, by the grants VEGA 1/0529/15 and VEGA 1/0908/15.

## Efficiency of GeoGebra Environment in Blended Learning of Functions

#### Vladimir Francisti

University of Novi Sad, Faculty of Sciences, Serbia

A modern approach of teaching mathematics based on the distance collaborative blended learning of function contents is not yet fully researched. Variety of technologies is being used for distance education to enhance interaction between students. The easiest and most accessible are internet educational websites and open source software like GeoGebra.

Internet educational software and programs are being used in supporting students is Serbia because it is most suitable, most affordable and has the ability to connect less privileged people to information. Despite evidence that educational software and programs can be used successfully as a cognitive delivery tool, the pedagogical approaches of educational software and programs have not yet been fully explored.

At the University of Novi Sad, Serbia, the collaborative learning has been used in calculus course, for examining functions and drawing their graphs for almost five years. In 2016 the authors decided to implement a new courseware calculus for learning functions concept to improve the collaborative learning that is accessible not only with a computer, but with every mobile phone with an internet connection.

The students in the control group have been studied the function concepts without the courseware calculus and with the use of education software GeoGebra at the University, and the students in the experimental group were using the new courseware calculus and GeoGebra freely and accessible at any time. We will be discussed visualization and dynamic character of the teaching with new courseware calculus and GeoGebra.

## Using Visualization in Combinatorics and Probability Tasks

#### Tadeáš Gavala

Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

According to many studies, visualization is helpful in solving combinatorics and probability problems. In this talk, we present selected combinatorics and probability tasks to illustrate the use of various visualization types to solve problems from this field of mathematics such as table, table of random numbers, schema, tree diagram, tangram, stochastic graph as polygon, stochastic graph as the process of the game, stochastic time graph, numeric axis, coordinate system, and picture. It turns out that introducing visualization during the task solving process can help students to understand the task solution and eliminate their misconceptions and thus be a tool that can be used also for the formative assessment.

Acknowledgement. The contribution has been created with the support of the project VEGA 1/0265/17 "Formative Assessment in Teaching of Natural Science, Mathematics and Informatics".

#### The Field $E_3$

#### Ján Haluška

#### Mathematical Institute SAS, Košice, Slovakia

The real line  $\mathbb{R}$  and the complex plane  $\mathbb{C}$  are Archimedean fields. We will show how to introduce a structure of a field in  $E_3$ . It will be equipped with an equivalence relation called the Cancellation law, known from physics.

Let us fix a multiplicative six-element Abel group  $A = \{i, j, k, I, J, K\}$ . The operation of multiplication is given by Latin square (where a splitting into two and two equal subsquares is denoted):

$\otimes$	i	j	k	I	J	K
i	i	j	k	Ι	J	$\overline{K}$
j	j	k	Ι	J	K	i
k	k	Ι	k I J	K	i	j
Ι	Ι	J	$egin{array}{c} K \\ i \\ j \end{array}$	i	j	k
J	J	K	i	j	k	Ι
K	K	i	j	$\mid k$	Ι	J

Starting from this group and the additive commutative semigroup  $(0, +\infty)$  we are able to define operations of multiplication and addition in  $E_3$ . Namely elements of A can be of sundry nature. The geometrical properties of A (symmetries and the dimension) are important. We may consider A as a set consisting of all vertices of the regular octahedron in the space  $E_3$ , i.e.,  $A = \{(1,0,0), (0,1,0), (0,0,1), (-1,0,0), (0,-1.0), (0,0,-1)\}$ . The imagination of octahedron can serve as a visualisation of addition and multiplication in  $E_3$  connected with the operation  $\otimes$  on A.

The centered regular 6-star polygon with vertices in the plane and with the same mathematics as above yields exactly a model of the Gaussian complex plane, i.e. the  $E_2$  with suitable topology and operations of addition and multiplication. Note, that in  $E_4, E_5, \ldots$  we have no regular polytops with six vertices and there are more of Latin squares with six rows and a unit element.

## On Real Functions and Monounary Algebras Emília Halušková

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We deal with the question of what the view to some real functions of one variable through monounary algebras gives. For example, if A is the monounary algebra of a monotone function, then the number of components of A is continuum and there are three connected monounary algebras only that are (up to isomorphisms) components of A.

Homomorphisms of monounary algebras make possible to solve some functional equations that are difficult for classical methods. We illustrate it by the equation 2f(x) - f(2x+1) + 1 = 0.

Acnowledgement. This research has been supported by Slovak Research Agency grant VEGA 2/0044/16.

- Chvalina, J., Kopeček, O., Novotný, M.: Homomorphic transformations - why and possible ways to how. Masaryk University, Brno, 2012.
- [2] Jakubíková-Studenovská, D., Pócs, J.: Monounary algebras. UPJŠ, Košice, 2009.
- [3] Halušková, E.: Homomorphic transformations as a method of approaching movement issues in systems. Proceedings of the conference "University as a place of dialogue", Verbum, Ružomberok 2014, 346–350 (in Slovak).

## On Three Classes of Real Functions Anton Hoyana

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Validity of integral inequalities for certain non-additive integrals has been firstly proved for two functions with the same monotonicity. This property of functions is usually called comonotonicity. However, the classical additive integral inequalities are free of this condition. We restrict our attention to two different concepts generalizing the comonotonicity condition. Namely, we follow an approach from [2], where authors introduced positive dependency of functions with respect to a (non-additive) measure m and a binary operator  $\triangle$ . This approach of m-positively dependent functions has helped the authors to prove a necessary and sufficient condition for validity of the Chebyshev type inequality. On the other hand, a different approach used in [1] provides a class of  $\star$ -associated functions for proving the validity of Minkowski-Hölder type inequalities. Our aim is to exemplify the classes of functions, compare them and find the conditions under which they become pairwise equivalent.

Acknowledgement. The support of the grants APVV-16-0337 and VVGS-PF-2017-255 is kindly announced.

- Boczek, M., Kaluszka, M.: On the Minkowski-Hölder type inequalities for generalized Sugeno integrals with an application. Kybernetika 52 (2016), 329–247.
- [2] Kaluszka, M., Okolewski, A., Boczek, M.: On Chebyshev type inequalities for generalized Sugeno integrals. Fuzzy Sets and Systems 244 (2014), 51–62.

## On Sums of Binomial Coefficients, Wavelets, Complex Analysis and Operator Theory

#### Ondrej Hutník

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Usually, three approaches are used to prove identities involving binomial coefficients as well as identities for sums of binomial coefficients:

- an algebraic proof it transforms one side of the equation with the aid of substitutions and of arithmetic operations into the expression on the other side;
- a combinatorial proof it shows that the expressions on both sides count the same things;
- a probabilistic proof it involves the computation of the probability of a certain event in two different ways and equating them.

However, a more challenging problem is to find a simple closed formula for an expression (such as the sums of combination coefficients). The talk will touch a hidden connection among the theory of wavelets, special functions, complex analysis and operator theory in order to evaluate certain sums involving binomial coefficients. Our approach is based on a relationship between spectral functions corresponding to Toeplitz operators acting on weighted Bergman spaces and Toeplitz operators acting on true-polyanalytic Bergman spaces over the upper half-plane. Choosing various generating symbols of Toeplitz operators we may prove many interesting identities for sums of binomial coefficients. Some concrete examples are provided.

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#### A Bound for Matrix Products in Special Case

## $\begin{array}{c} \mbox{Arthur Kennedy-Cochran-Patrick}^1, \mbox{Sergev}^1, \\ \mbox{and Štefan Berežn} \acute{y}^{1,2} \end{array}$

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We consider inhomogeneous matrix products over max-plus algebra, where the matrices in the product satisfy certain assumptions under which the matrix products of sufficient length be rank-one, as it was shown in [5]. We establish a bound on the transient after which this starts to happen for any product of matrices whose length exceeds that bound.

#### Main Results

**Definition 1** A digraph associated with a square matrix A is a digraph  $D_A = (N_A, E_A)$  where the set  $N_A$  has the same number of elements as the number of rows or columns in the matrix A. The set  $E_A \subseteq N_A \times N_A$  is the set of arcs in  $D_A$  where the weight of each arc is associated with the respective entry in the matrix A, i.e.  $w_{i,j} = a_{i,j} \in \mathbb{R}_{\text{max}}$ . If an entry in the matrix is negative infinity, this means that there is no arc connecting those nodes in that direction.

**Definition 2** The trellis digraph  $\mathcal{T}_{\Gamma(k)} = (\mathcal{N}, \mathcal{E})$  associated with the product  $\Gamma(k) = A_1 \otimes A_2 \otimes \ldots \otimes A_k$  is the digraph with the set of nodes  $\mathcal{N}$  and the set of edges  $\mathcal{E}$ , where:

- (1)  $\mathcal{N}$  consists of k + 1 copies of N which are denoted  $N_0, \ldots, N_k$ , and the nodes in  $N_l$  for each  $0 \le l \le k$  are denoted by  $1: l, \ldots, n: l$ ;
- (2) £ is defined by the following rules:
  a) there are edges only between N<sub>l</sub> and N<sub>l+1</sub> for each l,
  b) we have (i : (l − 1), j : l) ∈ £ if and only if (i, j) is an edge of D<sub>A<sub>l</sub></sub>, and the weight of that edge is (A<sub>l</sub>)<sub>i,j</sub>.

The weight of a walk W on  $\mathcal{T}_{\Gamma(k)}$  is denoted by  $p_{\mathcal{T}}(W)$ .

Here we will introduce the notation that will be used as follows theorem and corollaries:

Notation 1 The "boundaries" of  $\mathcal{X}$ :

- $A^{\sup}$ : the entrywise supremum over all matrices in  $\mathcal{X}$ . More precisely expression:  $A_{ij}^{\sup} = \sup_{X \in \mathcal{X}} (X)_{ij} = \bigoplus_{X \in \mathcal{X}} X$ . The weight of a walk W on  $\mathcal{D}_{A^{\sup}}$  will be denoted by  $p_{\sup}(W)$ .
- $A^{\text{inf}}$ : the entrywise *infimum over all matrices* in  $\mathcal{X}$ . More precisely expresion:  $A_{ij}^{\inf} = \inf_{X \in \mathcal{X}} (X)_{ij}$ .

Notation 2  $\lambda_2$ : the second largest cycle mean in  $\mathcal{D}_{A^{sup}}$ .

Notation 3 Weights of some paths and walks on  $\mathcal{D}_{A^{sup}}$  and  $\mathcal{D}_{A^{inf}}$ :

- $\alpha_i$ : the maximal weight of paths on  $\mathcal{D}_{A^{\sup}}$  connecting *i* to 1;
- $\beta_j$ : the maximal weight of paths on  $\mathcal{D}_{A^{\sup}}$  connecting 1 to j;
- $\gamma_{ij}$ : the maximal weight of paths on  $\mathcal{D}_{A^{\sup}}$  connecting *i* to *j* and not going through node 1.
- $w_i$ : the maximal weight of walks of length not exceeding k on  $\mathcal{D}_{A^{inf}}$  connecting i to 1;
- $v_j$ : the maximal weight of walks of length not exceeding k on  $\mathcal{D}_{A^{inf}}$  connecting 1 to j.

**Theorem 1** Let  $\Gamma(k)$  be an inhomogenous max-plus matrix product  $\Gamma(k) = A_1 \otimes A_2 \otimes \ldots \otimes A_k$  with k satisfying

$$k > \max\left(\frac{w_i + v_j - \gamma_{ij}}{\lambda_2} + (n-1), \frac{w_i - \alpha_i + v_j - \beta_j}{\lambda_2} + 2(n-1))\right) \quad (1)$$

for some  $i, j \in N$ , then

$$\Gamma(k)_{i,j} = \Gamma(k)_{i,1} \otimes \Gamma(k)_{1,j} = \Gamma(k)_{i,1} + \Gamma(k)_{1,j}.$$

If the maximum (1) is calculated over all  $i, j \in N$ , then we can write a matrix form, where  $\Gamma(k)$  is rank one and:

$$\Gamma(k) = (\Gamma(k)_{1,1}, \Gamma(k)_{2,1}, \dots, \Gamma(k)_{n,1})^{\top} \otimes (\Gamma(k)_{1,1}, \Gamma(k)_{1,2}, \dots, \Gamma(k)_{1,n}).$$

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#### References

- P. Butkovic: Max-linear Systems: Theory and Algorithms. In: Springer Monographs in Mathematics, London (2010).
- [2] B. Kersbergen: Modeling and Control of Switching Max-Plus-Linear Systems. In: Ph.D. Thesis, TU Delft, (2015).
- [3] G. Merlet T. Nowak S. Sergeev: Weak CSR expansions and transience bounds in max-plus algebra. In: Linear Algebra and its Applications 461 (2014), 163–199.
- [4] G. Merlet T. Nowak H. Schneider S. Sergeev: Generalizations of bounds on the index of convergence to weighted digraphs. In: Discrete Applied Mathematics 178 (2014), 121–134.
- [5] L. Shue B.D.O. Anderson S. Dey: On steady state properties of certain max-plus products. In: Proceedings of the American Control Conference, Philadelphia, Pensylvania, (June 1998), 1909–1913.

#### **3-Coloring of Claw-Free Graphs**

#### Mária Maceková and Fréderic Maffray

#### Institute of Mathematics FSc, Pavol Jozef Šafárik University, Jesenná 5, 041 54 Košice, Slovakia

Given an integer k, a k-coloring of a graph G is a mapping  $f: V(G) \rightarrow \{1, \ldots, k\}$  such that any two adjacent vertices  $u, v \in V(G)$  satisfy  $f(u) \neq f(v)$ . The chromatic number,  $\chi(G)$ , of G is the smallest integer k such that G admits a k-coloring. Vertex coloring is the problem of determining the chromatic number of a graph; it is a well-known NP-hard problem. In fact, even determining if a graph is 3-colorable is NP-complete and it remains NP-complete also in the class of claw-free graphs in general. In the talk we focus on the computational complexity of the problem in subclasses of claw-free graphs defined by forbidding additional subgraphs. We give a necessary condition for the polynomial-time solvability of the problem in such classes and present some new results.

## Uniform Approximation of the Eigenvalues of Hermitean Toeplitz Matrices

#### Egor A. Maximenko

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Let *a* be a continuous real-valued  $2\pi$ -periodic function defined on the real line,  $a_k$  (where  $k \in \mathbb{Z}$ ) be the *k*-st Fourier coefficient of *a*, and  $T_n(a)$  (where  $n \in \mathbb{N}$ ) be the  $n \times n$  Toeplitz matrix  $[a_{j-k}]_{j,k=1}^n$ . Denote by  $\lambda_{n,1}, \ldots, \lambda_{n,n}$ the eigenvalues of  $T_n(a)$ , written in the ascending order, and consider their asymptotic behavior, as *n* tends to infinity. It turns out that a first-order approximation of these eigenvalues can be given in very simple terms.

Let  $v_{n,1}, v_{n,2}, \ldots, v_{n,n}$  be the list obtained by writing the real numbers  $a(2\pi/n), a(4\pi/n), \ldots, a(2\pi)$  in the ascending order. We prove that  $|\lambda_{n,k} - v_{n,k}|$  tends uniformly to zero, as *n* tends to infinity. In other words, the eigenvalues of the Hermitean Toeplitz matrices, generated by a continuous function, can be uniformly approximated by the ordered values of the generating function at the uniform mesh.

Acknowledgements. This result is obtained jointly with A. Böttcher, J.M. Bogoya, and S.M. Grudsky. The talk is supported by the Erasmus+ mobility grant and partially by IPN-SIP projects.

## Unique-Maximum Coloring of Plane Graphs

#### Simona Rindošová

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Two edges of a plane graph are *facially adjacent*, if they are adjacent and consecutive in a cyclic order around their end vertex. *Facially proper edge* (*total*) coloring of a plane graph is a coloring in which every two facially adjacent edges (as well as every two adjacent vertices and every edge with

its end vertex) have different colors. Unique-maximum coloring of a plane graph is a coloring in which for each face the maximum color occurs exactly once on its elements (vertex or edge).

In this talk we deal with facially proper unique-maximum edge (total) coloring of plane graphs and their list versions and we present upper bounds on the corresponding chromatic numbers.

## Ideals and Selection Principle $S_1(\mathcal{P}, \mathcal{R})$

#### Viera Šottová

The main objects of our research are selection principles which have been introduced by M. Scheepers in [1, 2] where author has described the basic relations among different kinds of selection principles which are summarized in Scheepers' Diagram, see [1]. This area is so widespread and nowadays there have been many various results published so far.

We investigate ideal versions of Scheepers'  $S_1(\Gamma, \Gamma)$ -space, Arkhangelskii's  $\alpha_4$  property and Scheepers' monotonic sequence selection property, i.e.  $S_1(\mathcal{I}-\Gamma, \mathcal{J}-\Gamma)$ -space,  $S_1(\mathcal{I}-\Gamma_0, \mathcal{J}-\Gamma_0)$ -space and  $S_1(\mathcal{I}-\Gamma_0^m, \mathcal{J}-\Gamma_0)$ -space, respectively. We also show that cardinal invariant  $\lambda(\mathcal{I}, \mathcal{J})$  introduced in [3] is their common critical cardinality. Therefore, we are interested in this combinatorial characteristic as well.

Acknowledgement. The support of grant VEGA 1-0097-16 is kindly announced.

- Scheepers, M.: Combinatorics of open covers I: Ramsey theory, Topology Appl. 69 (1996), 31–62.
- [2] Scheepers, M.: C<sub>p</sub>(X) and Archangelskii's α<sub>i</sub>-spaces, Topology Appl. 89 (1998), 256–275.
- [3] Supina, J.: Ideal QN-spaces, J. Math. Anal. Appl. 434 (2016) 477–491.

## Some Remarks on Size-Based Integrals on Discrete Space

#### Jaroslav Šupina

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We discuss a concept of size-based integral on discrete space and its perspective role in decision making applications. The concept of size and corresponding super level measures were introduced by Y. Do and C. Thiele [2] in harmonic and time-frequency analysis context. The talk is based on joint results with J. Borzová, L. Halčinová, O. Hutník and J. Kiseľák [1, 3].

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- Borzová, J., Halčinová, L., Šupina ,J.: Size-based super level measures on discrete space, (accepted).
- [2] Do, Y., Thiele, C.: L<sup>p</sup> theory for outer measures and two themes of Lennart Carleson united, Bull. Amer. Math. Sci. 52 (2) (2015), 249– 296.
- [3] Halčinová, L., Hutník, O., Kiseľák, J., Šupina, J.: Beyond the scope of super level measures, Fuzzy Sets and Systems, https://doi.org/10. 1016/j.fss.2018.03.007.

## Graph Cutting in Image Processing and Its Applications

#### Mária Ždímalová

Department of Mathematics and Constructive Geometry, Slovak University of Technology in Bratislava, Slovakia

Image processing includes many tools with handling of images as well as work with different types of data representing like images. Segmentation is one part of image analysis. In this contribution we focuse on segmentation techniques from graph theory approach. We will introduce the method of "graph cutting" and "intelligent scissors" and its application with artificial as well as real data, as medical, bio-medical data, SAT images. The main idea of the "graph cutting" is representing the image like a graph, and a network creation from the image. In this network we apply the algorithm for finding maximum cut in the network (Ford Fulkerson algorithm, Prim's algorithm), which corresponds to a minimal cut in the network. It is well known that it corresponds to a problem like: finding minimal cut is the same problem like finding segmentation in the picture. In "intelligent scissors" we use an optimization of the Dijsktra algorithm. We use, optimize and apply this technique for different types of real data. We created own common program and within a collaboration with Medical Faculty of Comenius University in Bratislava.

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#### Program 19. Konferencie košických matematikov

#### Programme of the 19<sup>th</sup> Conference of Košice Mathematicians

Štvrtok – Thursday 12. 4. 2018

#### $12^{30}$ – **Obed** – Lunch

#### $13^{55}$ – Otvorenie konferencie – Conference opening

- 14<sup>00</sup> Erika Fecková Škrabuľáková (ÚRaIVP FBERG TU) On an Optimal Distributions of Issues and Costs Reducing via Graph Algorithms
- 14<sup>20</sup> Mária Maceková (ÚMV PF UPJŠ) 3-Colorability of Claw-Free Graphs
- 14<sup>40</sup> Simona Rindošová (ÚMV PF UPJŠ) Facial Unique-Maximum Coloring of Plane Graphs
- $15^{00}$  Štefan Berežný (KMTI FEI TU) A Bound for Matrix Products in Special Case
- 15<sup>20</sup> Vladimir Francisti (FoSc University of Novi Sad, Serbia) Efficiency of Geogebra Environment in Blended Learning of Functions

#### 15<sup>40</sup> – Občerstvenie – Coffee-break

- 16<sup>00</sup> Egor A. Maximenko (ESFM IPN México) Uniform Approximation of the Eigenvalues of Hermitean Toeplitz Matrices
- 16<sup>20</sup> Ondrej Hutník (ÚMV PF UPJŠ) On Sums of Binomial Coefficients, Wavelets, Complex Analysis and Operator Theory
- 16<sup>40</sup> Tadeáš Gavala (ÚMV PF UPJŠ) Využitie vizualizácie v úlohách z kombinatoriky a pravdepodobnosti
- $17^{00}$  Viera Šottová (ÚMV PF UPJŠ) Ideals and Selection Principle  $S_1(\mathcal{P}, \mathcal{R})$
- 17<sup>20</sup> Anton Hovana (ÚMV PF UPJŠ) On Three Classes of Real Functions
- $18^{00}$  Večera Dinner

#### Piatok - Friday 13. 4. 2018

- $9^{00}$  Borut Lužar (FIS Novo mesto Slovenia) Colorful Graph Theory
- 9<sup>50</sup> Vladimír Baláž (ÚIAM FCHPT STU Bratislava) On Generalized Notion of Convergence by Means of Ideal and Its Applications

#### $10^{40}$ – Občerstvenie – Coffee-break

11<sup>10</sup> – Ján T. Tóth (KMI EkF Univerzita J. Selyeho Komárno) On Partial Limits of Sequences

#### $12^{00}$ – **Obed** – Lunch

- 13<sup>30</sup> Stanislav Jendrol (ÚMV PF UPJŠ) Mathematics, Chemistry and Nobel Prizes
- 14<sup>20</sup> Vasile Berinde (NUC Baia Mare Romania) Pompeiu-Hausdorff Metric and Its Wide Spreading Role in Science and Technology

#### $15^{10}$ – Občerstvenie – Coffee-break

- 15<sup>40</sup> František Staněk (VŠB Ostrava) Methodology and Algorithms for Creating and Visualization of 3D Models for Mineral Deposits of Critical EU Commodities
- 16<sup>30</sup> Jiří Demel (SvF ČVUT Praha) Generování cvičných úloh
- 17<sup>20</sup> Monika Dillingerová (FMFI UK Bratislava) Logika na prijímacích pohovoroch na univerzity

#### 18<sup>30</sup> – Večera a spoločenský večer – Dinner & Party

#### Sobota - Saturday 14. 4. 2018

- 8<sup>40</sup> Jaroslav Šupina (ÚMV PF UPJŠ) Size Based Integrals
- 9<sup>00</sup> Emília Halušková (MÚ SAV Košice) O riešení niektorých typov funkcionálnych rovníc pomocou homomorfizmov monounárnych algebier
- 9<sup>20</sup> Ján Haluška (MÚ SAV Košice) Pole  $E_3$
- $9^{40}$  Mária Ždímalová (SvF STU Bratislava) Graph Gutting in Image Processing
- 10<sup>00</sup> Jozef Doboš (ÚMV PF UPJŠ) A View into the Workbooks and Textbooks of Mathematics
- 10<sup>20</sup> Mária Demlová (FEL ČVUT Praha) New Rules for assessing Mathematical Competencies – Erasmus+ project
- 10<sup>40</sup> Záver konferencie Conference closing
- $10^{45}$  Občerstvenie Coffee-break
- $11^{00} \mathbf{Obed} \mathbf{Lunch}$

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