

**Jednota slovenských matematikov a fyzikov
Pobočka Košice**

**Prírodovedecká fakulta UPJŠ
Ústav matematických vied**

**Fakulta elektrotechniky a informatiky TU
Katedra matematiky a teoretickej informatiky**

14. Konferencia košických matematikov

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Predhovor

Milí priatelia,

vítame vás na 14. Konferencii košických matematikov, tohto roku konanej pri príležitosti 50. výročia založenia Prírodovedeckej fakulty UPJŠ. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, Centrom excelentnosti informatických vied a znalostných systémov UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice v Herlanoch.

Cieľom konferencie je zintenzívniť stavovský život všetkých, ktorí sa v Košiciach a okolí profesionálne zaoberajú matematikou (t. j. učiteľov všetkých typov škôl, pracovníkov na poli matematických a informatických vied a aplikácií matematiky v priemysle, technike, bankovníctve a inde) a formulovať základné oblasti ich stavovských záujmov. Odborný program konferencie tradične pozostáva z pozvaných prednášok, prihlásených referátov a diskusií o stavovských problémoch. Na konferencii je vytvorený priestor aj na diskusiu o aktuálnych problémoch.

Priestor na získanie skúseností pri prezentácii svojich výsledkov je poskytnutý aj doktorandom a mladším matematikom. Je potešujúce vidieť, ako sa každým rokom zlepšujú ich vystúpenia. Veríme, že im vystúpenia na tejto konferencii pomôžu pri prezentovaní výsledkov na ďalších konferenciách.

Organizačný výbor konferencie pozýva významné osobnosti matematiky, ktoré v rámci svojich prednášok ukážu miesto matematiky v spoločenskom živote a súčasné trendy jej rozvoja. Viaceré z týchto prednášok mali taký pozitívny ohlas, že ich autori boli pozvaní predniesť ich aj na iných konferenciách. Toho roku pozvanie prednášať prijali: doc. RNDr. V. Bálint, CSc., mim. prof. (FPaEDaS Žilina), doc. RNDr. J. Brincková, CSc. (FPV UMB Banská Bystrica), prof. W. Forys (FMCSICS JU Kraków), doc. J. Chaparova (CAMI RU, Ruse, Bulharsko), doc. Mgr. P. Kovář, PhD. (VŠB TU Ostrava), doc. Mgr. R. Mařík, PhD. (ÚM MeU Brno), RNDr. M. Molnárová, PhD. (FEI TU Košice), doc. RNDr. M. Ploščica, CSc. (PF UPJŠ Košice), RNDr. L. Révészová, PhD. (EkF TU Košice) a RNDr. M. Saniga, DrSc. (AÚ SAV Tatranská Lomnica).

Prajeme vám príjemný pobyt v Herlanoch

Organizačný výbor: Ján Buša
Stanislav Jendroľ
Štefan Schrötter

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Invited lectures

Igor Kluvánek's Dream Remained a Dream

Vojtech Bálint

Žilinská univerzita, Univerzitná 1, 010 26 Žilina

In the first half of the 60-ies Igor Kluvánek worked hard on increasing significantly the professional level of teachers of mathematics in primary and secondary schools. Unfortunately, the level is today much lower and this state is a result of sometimes not very successful social changes. I will illustrate this with several examples.

School Logic – a Game of Codes and Ciphers

Jaroslava Brincková

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The set of concepts and ideas mastered by a pupil is denoted by the term *knowledge*. The comprehension of given ideas and concepts by pupils of the same class occurs in two different ways: *procesual* and *conceptual* perception and thinking. Many terms (concepts) in mathematics were developed from mathematical activities (processes). To acquire the generalization of the concept and its classification in a particular class, a symbol is necessary. The symbol is a unity of a material carrier and mental carrier of the meaning. Both parts constitute a bilaterality of symbol and are inseparably associated.

10¹⁵ – Filip Krajník (ÚMV PF UPJŠ) *Description of Congruence Lattices in CIP Varieties via Priestley Duality*

10⁴⁰ – Katarína Furcoňová (ÚMV PF UPJŠ) *Interactive Pen as a New Technological Aid*

12⁰⁰ – **Obed – Lunch**

14⁰⁰ – Erika Škrabuľáková (ÚRaIVP FBERG TU) *On the Facial Non-Repetitive List Vertex-Colouring of Graphs*

14²⁵ – Peter Hudák (ÚMV PF UPJŠ) *Properties of Subgraphs of 4-critical Planar Graphs*

14⁵⁰ – Mária Kubíková (ÚMV PF UPJŠ) *Facial Complete Vertex Colouring for Some Types of Plane Graphs*

15¹⁵ – Michaela Vrbjarová (ÚMV PF UPJŠ) *Complete Edge Colorings of Plane Graphs*

15⁴⁰ – **Občerstvenie – Coffee-break**

16⁰⁵ – Pavol Široczki (ÚMV PF UPJŠ) *Mycielskian and the Graph Dimension*

16³⁰ – Martina Mockovčiaková (ÚMV PF UPJŠ) *Strong Edge Coloring of Bipartite Graphs*

16⁵⁵ – Peter Šugerek (ÚMV PF UPJŠ) *ℓ-Facial Edge Coloring of Plane Graphs*

17²⁰ – Anna Polomčáková (ÚMV PF UPJŠ) *Teachers' Attitudes to Teaching Combinatorics*

18⁰⁰ – **Večera – Dinner**

19⁰⁰ – **Workshop:** Robert Mařík – *Technical and T_EXnical Background of the System MAW (Mathematical Assistant on Web)*

Piatok – Friday 5. 4. 2013

8³⁰ – Miroslav Ploščica (ÚMV PF UPJŠ) *Computation of Galois Groups*

9²⁰ – Metod Saniga (AÚ SAV Tatranská Lomnica) *Finite Geometries with a Quantum Physical Flavour*

Program 14. Konferencie košických matematikov

Programme of the 14th Conference of Košice Mathematicians

Streda – Wednesday 3. 4. 2013

12³⁰ – Obed – Lunch

14¹⁵ – Otvorenie konferencie – Conference opening

14²⁰ – Jaroslav Šupina (ÚMV PF UPJŠ) *Sequences of Sequences of Functions*

14⁴⁵ – Michal Dečo (ÚMV PF UPJŠ) *Strongly Dominating Sets of Reals*

15¹⁰ – Jana Chudá (ÚMV PF UPJŠ) *Motion Word Problems*

15³⁰ – Noémi Székelyová (ÚMV PF UPJŠ) *Procedure, Process, and Concept*

16⁰⁰ – Občerstvenie – Coffee-break

16³⁰ – Martina Ivanecká (ÚMV PF UPJŠ) *Students' Conceptual and Procedural Knowledge of Some Statistical Concepts*

16⁵⁵ – Katarína Kocová Mičkaninová (ÚMV PF UPJŠ) *Modern Digital Technologies in the Teaching of Probability*

17²⁰ – František Mošna (ČZU Praha) *Hypotheses Testing and Possibilities of e-Learning*

18⁰⁰ – Večera – Dinner

Štvrtok – Thursday 4. 4. 2013

8³⁰ – Lenka Halčinová (ÚMV PF UPJŠ) *On Integration with Respect to a Probabilistic Measure*

8⁵⁵ – Jana Molnárová (ÚMV PF UPJŠ) *On Universal Integrals*

9²⁰ – Zuzana Farkasová (ÚMV PF UPJŠ) *About Polymorphism-Homogeneous Monounary Algebras*

9⁴⁵ – Občerstvenie – Coffee-break

An inseparable part of a concept is a **word**. It forms material cover of a term. It represents generalization, classes of phenomena. The conceptual thinking cannot exist without verbal thinking. A word is often terminated by its ratio to the concept. We can represent the schematical qualification of communication flow, from the presentation to registration of information, in the diagram:

$$\begin{array}{ccccccc} \text{sender} & \longrightarrow & \text{encoding} & \longrightarrow & \text{transmission} & \longrightarrow & \text{decoding} & \longrightarrow \\ & & & & & & & \longrightarrow \text{receiver} & \longrightarrow & \text{coding} \end{array}$$

There can be various relations between the symbol and the labelled object. We refer to the relation of similarity or logical connection as a 'motivated symbol'. In the case of the purely incidental relation, we use the term 'conventional' or 'arbitrary symbol'.

One of the conditions of acquiring the subject matter on logic and sets at secondary level of education is the ability to encode the sentences using symbols and the ability to decode the mathematical notations to Slovak language. Card games, in which combinatorial thinking is applied, are suitable for the development of ability to work with ciphers and logical thinking of 10-year-old pupils. One of the successful games is a SET game. In our talk, we present the mathematical background and the application of the SET game to Logic lessons.

Symbolic Dynamics of Parallel Processes

Wit Foryś

Uniwersytet Jagiellonski, Kraków, Poland

The aim of my talk is to present a framework for a systematic research of symbolic dynamics of concurrently evolving systems. The idea of the framework is to join symbolic dynamics and trace theory. Notice, that these two bases and all problems as well, which we have to face are of the dual nature. Namely combinatorial (combinatorics on words) and topological.

Some Ideas of Geometry of Curves and Surfaces Illustrated by Mathematica

Julia V. Chaparova

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The computer algebra system Mathematica is provided with powerful tools for symbolic and numerical calculations that can be used both in elementary or scientific problems, and it possesses valuable resources to produce 2D and 3D graphics. The purpose of the article is to present examples concerning geometry of curves and surfaces using Mathematica. Included are examples of parameterized curve and tangent vector field, curve on surface and the Frenet frame, tangent plane to a surface.

In the study of systems of ordinary differential equations the vector field of the system is the major tool to determine the qualitative behavior of the solution set and to have a notion of the phase portrait. And here the computer algebra systems must take into account. Next, we consider examples of linear and nonlinear systems of ordinary differential equations, and discuss questions of equilibrium points, stability, solution set and phase portrait.

Fair Incomplete Tournaments

Petr Kovář

VŠB – Technical University of Ostrava, Czech Republic

In a round robin tournament of n players or teams meets each player every other opponent in one game. If there is not enough time to play all the games during one season, which can easily happen in tournaments with twenty or more players, the organizers have to choose which opponents will meet and which not. Naturally, one expects that the selection will be fair in the following sense. It should not happen that some teams will play only strong opponents while other teams play only weak opponents.

We use graph theory to describe the tournament and we introduce a family of related graph labelings that can be used to schedule fair incomplete tournaments.

Sequences of Sequences of Functions

Jaroslav Šupina

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M. Scheepers [2] and D.H. Fremlin [1] proved that selection property for sequences of sequences of continuous functions considered by several authors can be described by the property related to quality of convergence of sequences of continuous functions. We present ideas of their proofs and we show how to modify Fremlin's proof to obtain the result for various classes of real-valued functions on a topological space.

Acknowledgement. Partially supported by the grant 1/0002/12 of Slovenská grantová agentúra VEGA.

References

- [1] Fremlin D.H.: SSP and wQN, Notes of 14. 01. 2003, <http://www.essex.ac.uk/math/people/fremlin/preprints.htm>.
- [2] Scheepers M.: *Sequential Convergence in $C_p(X)$ and a Covering Property*, East-West J. of Mathematics **1** (1999), 207–214.

Complete Edge Colorings of Plane Graphs

Michaela Vrbjarová

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The complete edge coloring is a proper edge coloring in which any two colors assigned to edges of a graph meet on two adjacent edges and the facial complete edge coloring is the restriction of complete edge coloring on graph's faces of plane graphs. The maximum number of colors, which can be assigned to edges of the graph so that coloring was complete edge coloring is called the achromatic index and the maximum number of colors, which can be assigned to edges of the graph so that coloring was facial complete edge-coloring is called facial achromatic index.

We determine or at least find estimates of the achromatic index and the facial achromatic index for some special classes of graphs.

On the Facial Non-Repetitive List Vertex-Colouring of Graphs

Erika Škrabuláková

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Non-repetitive sequences are the sequences where no two consecutive blocks are the same. By Alon, Grytczuk, Hałuszczak and Riordan non-repetitive sequences were introduced also into the graph theory: The vertex-colouring $\phi : V(G) \rightarrow \{1, 2, \dots, k\}$ of a graph G is *non-repetitive* if the sequence of colours on any path in G is non-repetitive. As the problem to find the minimum number of colours that can be used to colour the vertices of arbitrary graph such that the obtained colouring is non-repetitive seems to be very hard to be solved in general, a lot of variations of the original problem occurred in the scientific literature. One of them is the relaxation where we do not ask for the condition that every path in a graph G has to be non-repetitive but only every *facial path* - path made of consecutive vertices and edges on the boundary walk of some face, has to fulfill this condition: Let G be a plane graph. A *facial non-repetitive vertex-colouring* of G is a vertex-colouring of G such that any facial path is non-repetitive. Sometimes it is not possible to use arbitrary colour to colour every vertex of a given graph but only a colour from some list preassigned to this vertex. In such a case we are speaking about a list colouring of a graph.

A list vertex-colouring where only facial paths of a plane graph G are required to be coloured non-repetitively is a *facial non-repetitive list vertex-colouring* ϕ_{fl} of a graph G , where $L : V(G) \rightarrow 2^{\mathbb{N}}$ is a list assignment of G . If such a colouring exists for any list assignment L with minimum list length at least k , we call G *facial non-repetitive k -vertex-choosable*. The minimum number k such that G is facial non-repetitive k -vertex-choosable is the *facial Thue choice number* of G and we denote it by $\pi_{fl}(G)$. We determine this parameter for several classes of graphs as well as give a general upper bound for it.

Simpsons Math

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Have Simpsons and math something in common? And how about Simpsons and mathematicians?

Different Faces of Mathematics (Joyful, Playful, Free, Interactive, Automatic)

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Nowadays on the Internet there are many teaching materials, covering almost all levels and areas of the education. As a consequence of this giant supply, the students as consumers of these materials have very high expectations. Despite the fact that professional mathematicians consider the content of textbooks to be far more important than the look and design, the young generation often holds the opposite opinion. For this reason it is therefore important to enrich the teaching and learning materials by parts which attract interest to the problem. This talk will present some techniques for including such parts into static eLearning materials .

We will present AcroTeX: a system for preparing interactive mathematical tests. These tests help students to check their own knowledge and understanding of the topic. We will also introduce on-line system Mathematical Assistant on Web. This system is capable to simulate human approach to solving some types of mathematical problems and automatically elaborate the sample solution. Further, we show that it is possible to prepare mathematical versions of knowledge games known in Czech television under the names Riskuj (Jeopardy) and AZ quiz.

We will provide examples of finished materials as well as links and instructions how to create your own interactive materials. An important issue is the fact that all the materials presented on the talk can be build using free software.

Some parts of the talk will include presentation of materials developed as parts of projects “Matematika s radostí” (Mathematics with joy) and “Akademie” (Academy) supported by the European Structural Fund, Operational Program Education for Competitiveness.

Technical and T_EXnical Background of the System MAW (Mathematical Assistant on Web)

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At the workshop we briefly introduce a system Mathematical Assistant on Web. It is a set of web sheets/forms available on <http://user.mendelu.cz/marik/maw> intended for automatic solution of some types of problems usually included within mathematical courses at the technical universities. It is concerned, e.g., on calculus – derivatives, integrals, line integrals, solution of differential equations of the first and second order, and other. The output produced by the system is not only the final result, but also automatically generated technique of the solution including important intermediate steps and particular results. After the entrance system introduction it will be possible to discuss its technical background, to check its functionality, or to share experiences with its operating and using.

The Robustness of Interval Fuzzy Matrices

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The fuzzy algebra \mathcal{B} is a triple (B, \oplus, \otimes) , where (B, \leq) is a bounded linearly ordered set with binary operations *maximum* and *minimum*, denoted by \oplus, \otimes . Matrices with inexact data (interval matrices) over fuzzy algebra are studied.

Procedure, Process, and Concept

Noémi Székelyová

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Procedure, process and concept are key terms in the didactics of mathematics. We use the term procedure as a specific algorithm for implementing a process and the term process in a more general sense as the cognitive representation of mathematical operation. By concept we mean an idea stored in the mind. The goal of this contribution is to further explain these terms in the context of mathematics education. We also want to support the explanation with examples of tasks and pupils' solutions.

Mycielskian and the Graph Dimension

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The dimension of a graph G is the smallest integer n such that G has an unit distance representation in \mathbb{R}_n (with the standard Euclidean distance). We explore the relation between the dimensions of a graph and its Mycielskian, and describe the cases where both dimensions are small.

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used in this topic, as well as the attitude of students to combinatorics from a teachers' point of view. According to previous research teachers do not tend to have a positive approach to teaching combinatorics. This result was not confirmed by this research.

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Data Encryption and Prime Numbers

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Data encryption is an important part of digital communication. Today, nearly 90% of all the computational power is used to data encryption. Prime numbers play an important role in encrypted digital communication.

The work includes a brief history of primes and examines their relationship to the encryption algorithm. Currently, prime numbers, data encryption and information technology allow a new way of proof, i.e. interactive proof. We use interactive proofs in everyday life, for example, in the verification process of identity by using credit cards. The work also describes the principles and some applications of interactive proofs.

Key words: *data encryption, prime numbers, interactive proofs*

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Equivalent conditions for interval fuzzy matrices to be possibly and universally robust, respectively, are presented. Polynomial algorithms for checking the necessary and sufficient conditions for robustness of interval fuzzy matrices are described. In addition, more efficient algorithms for verifying the robustness of interval circulant matrices are introduced.

Acknowledgements. This work was supported by the Slovak Cultural and Education Agency No. 032 TUKE-4/2013.

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Computation of Galois Groups

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The Galois group of a polynomial f over a field F is the group of all automorphisms of its splitting field, which preserve F . A well known theorem says that if f over \mathbb{Q} is solvable in radicals, then the Galois group of f is solvable.

Our lecture is devoted to the problem of determining of the Galois group of a given polynomial. This is a quite nontrivial task, whose computational complexity increases rapidly with rising degree of f . We concentrate mainly on the method of invariants – polynomials of several variables that are invariant under some permutations of their variables. We present several general results, for instance a criterion determining whether the Galois group is a subgroup of the alternating group. We provide a detailed analysis of the situation for polynomials of low degrees.

We also discuss several related problems, for instance the classification of transitive permutation groups and decomposition of polynomials over finite extensions of \mathbb{Q} .

Information and Computer Literacy of Secondary School Graduates

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“Your ability to gain, process and use information decides whether you will belong to winners or losers.” wrote Bill Gates in 1999. Since then, the meaning of his words has been constantly increasing. A new environment of the knowledge society is creating under the influence of penetration of information and communication technologies (ICT) in all areas of life, and also due to the emerging knowledge technologies. We often recognize an attitude that the primary things in development, implementation and effective application of ICT are modern technical devices, computers – hardware and software. These are surely unthinkable part of it, however the key part are people – users. Firstly, it depends on competent users what they will expect from ICT, information systems and informatics, what will be their requirements on information, its quality and quantity. The investments into informatization projects and new technology are often very high. The efficiency of these investments depends on users, their motivation, interest and proper education. It is obvious that support and development of information and computer literacy of all level users should be the main interest. It is obvious as well, that the system of education has to be customized according to these social requirements. The required state and the reality however do not have to meet. The lecture summarizes results of the research of information and computer literacy of secondary school graduates

same task in his bachelor thesis. The main difference between these two tools is that the first one is working on the side of server while the second one is working on the side of client. Both can be implemented to HTML code.

During my work on these web-sites I used especially the switches, which enable branching of individual parts of them, and forms and their elements from language HTML. One complication occurred when I studied the possibilities of calculating values of distribution functions or critical values. Neither PHP nor databases manager MySQL (which cooperates with PHP) does not contain required statistical functions and does not support any other possibility to get them. In the end I used the simple cooperation of PHP with ordinary text files and this web-sites gain the critical values from something like statistical tables. The weakness of such attitude is that the level of test cannot be arbitrary but only 0,05 or 0,01.

The experience with these web-sites is outstanding. Students can focus on the main ideas of statistical testing, they can change data during their work and compare results, they can perform various experiments with them. At last but not least these web-sites can help the teacher to create more interesting statistical examples during his preparation for teaching, too.

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Teachers' Attitudes to Teaching Combinatorics

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In this contribution we evaluate our research aimed at teaching of combinatorics at Slovak schools. The questions were focused on the opinions and mindset of teachers on teaching combinatorics, textbooks and methods

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Hypotheses Testing and Possibilities of e-Learning

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Mathematics and statistics even more are the less popular subjects at school. Teaching of statistics at secondary school is usually reduced only to the calculation of the mean or the standard deviation. One of the main principles of mathematical statistics – hypotheses testing – is presented to students first at universities. Even here the greatest interest is focused rather on calculation according to the given formulas than to the basic ideas of this method or evaluation, mathematization and interpretation of real situations.

So I was looking for some tool which could simplify and automate statistical calculation and hence bring more attention on core of hypotheses testing. There are some tools in Excel or Mathematica and plenty of special statistical programmes, e.g., Statistica, SPSS, SAS, Statgraphic, R etc. But for using such tools students need to acquire certain skills how to handle with them. Even on web (with some exceptions) I did not find any appropriate tools.

That is why I decided to create special web-sites which could simplify the process of calculation and at the same time they are easily manageable and accessible.

The main idea of such web-sites is as follows. User can enter range of random sample at first. Then the required number of items is opened and user can enter data. After submitting data, the calculation is performed and the result is stated. There are several technical possibilities how to create such web-sites. According to my opinion the most useful of them is the programme PHP. I chose this tool for my work. My student Daniel Mbuy Lubanda from Kongo tries to use the programme JavaScript for the

who entered the first grade at the Faculty of Economics Technical University of Košice. We compared the results of questionnaires from years 2003–2005 and 2009–2012 to describe situation and changes in this area.

Finite Geometries with a Quantum Physical Flavour

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The talk will be an overview of a variety of finite geometries that have recently found interesting physical applications. We shall be concerned with three prominent classes of finite geometries, namely: 1) projective ring lines behind generalized Pauli groups of single-qudits, 2) symplectic polar spaces underlying the commutation algebra of generalized Pauli groups of multiple-qubits, and 3) finite generalized polygons underpinning certain stringy black-hole entropy formulas.

As per item one, the most important finding is the fact that *single* qudits distinguish whether they live in the Hilbert space of a square-free dimension, or not. Namely, there exists a bijection between the pairs/vectors (a, b) of the modular ring Z_d and the elements of the generalized Pauli group of the d -dimensional Hilbert space generated by the standard shift (X) and clock (Z) operators, $\propto X^a Z^b$. Under this correspondence, the operators of the group commuting with a given operator form the set-theoretic union of the points of the projective line over Z_d which contain a given pair if d is a product of distinct primes, and the span of the points of the line for any other values of d .

Concerning item two, here as the most crucial finding we can regard the fact that the algebra of the Pauli operators on the Hilbert space of N -qubits is embodied in the geometry of the symplectic polar space of rank N and order two, $W(2N - 1, 2)$. The operators (discarding the identity) answer to the points of $W(2N - 1, 2)$, their partition into maximally commuting subsets correspond to spreads of the space, a maximally commuting subset has its representative in a maximal totally isotropic subspace of $W(2N - 1, 2)$ and, finally, “commuting” translates into “collinear” (or “perpendicular”).

Under item three go two key findings. First is related to the E_7 -symmetric black hole entropy formula where the three-qubit Pauli group with its associated split Cayley hexagon of order two turned out to give a clear

understanding of an automorphism of order 7 relating the seven STU subsectors of $N = 8$, $D = 4$ supergravity and the explicit appearance of a discrete $PSL(2, 7)$ symmetry of the entropy formula — which is the automorphism group of the Coxeter graph sitting in the hexagon. A more profound discovery was showing that the E_6 -symmetric entropy formula describing black holes and black strings in $D = 5$ is intimately tied to the geometry of the generalized quadrangle $GQ(2, 4)$. The 27 charges correspond to the points and the 45 terms in the entropy formula to the lines of $GQ(2, 4)$. Different truncations with 15, 11 and 9 charges are represented by three distinguished subconfigurations of $GQ(2, 4)$; these are the $GQ(2, 2)$ with 15 points, the perp-set of a point with 11 points, and the $GQ(2, 1)$ with nine points, respectively.

We shall conclude the talk by a couple of interesting mathematical observations. The first is the existence of finite rings with unity featuring also free cyclic submodules generated by non-unimodular vectors, the smallest such ring being the non-commutative ring of order 8; this ring is remarkable in that the “non-unimodular” part of the projective plane defined over it features the ordinary Fano plane as the core geometry. The second one is recognition of the importance of the concept of the Veldkamp space, and how this concept sheds a novel light on the relation between $GQ(2, 4)$ and $GQ(4, 2)$.

On Universal Integrals

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The Choquet and the Sugeno integral provide a suitable tool for solving many problems connected with aggregation of data in engineering and social sciences. The construction of the Choquet integral is based on standard arithmetic operations of addition and multiplication, therefore each Choquet integral-based aggregation is co-monotone additive. On the other hand, the Sugeno integral is based on the lattice operations of join (maximum) and meet (minimum), thus the corresponding aggregation is join-or meet-homogeneous. Because of the used operations in construction of both the integrals, their applicability is restricted. This fact motivated the paper [1], where a construction of universal integral generalizing both the Choquet and the Sugeno case is presented. This new concept of integral is universal in the sense that it allows to integrate non-negative real-valued functions with respect to arbitrary (non-negative) monotone set functions in such a way that covers both the Choquet (1), the Sugeno integral (2) and also the Shilkret integral (3) which can be regarded as an ancestor of the Sugeno integral.

$$\mathbf{Ch}(m, f) = \int_0^\infty m(\{f \geq t\}) dt \quad (1)$$

$$\mathbf{Su}(m, f) = \sup_{t \in [0, \infty]} \{\min(t, m(\{f \geq t\}))\} \quad (2)$$

$$\mathbf{Sh}(m, f) = \sup_{t \in [0, \infty]} \{t \cdot m(\{f \geq t\})\} \quad (3)$$

Universal integrals are related to general operations of pseudo-multiplication and pseudo-addition. A different approach to construction of universal integrals is based on copulas and can be perceived as an expression of dependence of values of integrable functions and values of measure with respect to which we integrate. Certain construction methods and properties of the universal integrals will be discussed in our talk. Also, we will investigate convergence of universal integrals and provide new partial results.

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Strong Edge Coloring of Bipartite Graphs

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A *strong edge coloring* of a graph G is a proper edge coloring in which each color class is an induced matching of G ; that is, there is no bichromatic path of length three in G . The minimum number of colors of a strong edge coloring of G is called the *strong chromatic index* of G and denoted by $\chi'_s(G)$.

In 1993 Brualdi and Quinn conjectured that every bipartite graph with bipartition X and Y without any cycle of length four such that the maximum degree of any vertex in X is two and the maximum degree of any vertex in Y is Δ can be strongly colored with $\Delta + 2$ colors. We prove that this conjecture is true for such graphs with $\Delta = 3$.

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Conference contributions

Strongly Dominating Sets of Reals

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We analyze the structure of strongly dominating sets of reals introduced in [1]. In particular, we prove that for every $\kappa < \mathfrak{b}$ a κ -Suslin set $A \subseteq {}^\omega\omega$ is strongly dominating if and only if A has a Laver perfect subset.

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About Polymorphism-Homogeneous Monounary Algebras

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We say that a monounary algebra \mathcal{A} is homomorphism-homogeneous if every homomorphism from a finitely generated subalgebra of \mathcal{A} into \mathcal{A} extends to an endomorphism of \mathcal{A} . A k -ary local polymorphism of a given monounary algebra \mathcal{A} is a homomorphism from a finitely generated subalgebra of the power \mathcal{A}^k to \mathcal{A} . \mathcal{A} is said to be k -polymorphism-homogeneous if each k -ary local polymorphism can be extended to a global polymorphism of \mathcal{A} , and \mathcal{A} is called polymorphism-homogeneous, if it is k -polymorphism-homogeneous for each positive integer k .

We show that polymorphism-homogeneous monounary algebras can be described by the notion of homomorphism-homogeneity and we characterize all polymorphism-homogeneous monounary algebras.

Interactive Pen as a New Technological Aid

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Very actual theme in education is the integration of new technological aids. In this, we deal with advantages and disadvantages of using graphics tablet in teaching, which is a part of didactical technics.

Further we introduce an interactive pen that we see as natural and useful technological aid in analyzing of student's solutions of mathematical problems.

Facial Complete Vertex Colouring for Some Types of Plane Graphs

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We will consider some special vertex colouring of plane graphs called facial complete vertex colouring. It is such a proper vertex colouring that on any face between each pair of colours used on this face there is a pair of adjacent vertices coloured with these colours. The maximum number of colours for which there is a facial complete vertex colouring is called the facial achromatic number $\psi_f(G)$ of graph G . The paper describes the problem of existence of given colouring and it includes techniques how to construct this type of colouring and also values and estimates of $\psi_f(G)$ for selected types of planar graphs.

l -Facial Edge Coloring of Plane Graphs

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We consider a plane graph G . In an l -facial coloring, any two different vertices that lie on the same face α of G and are at distance at most l on α receive distinct colors. Král, Madaras, Škrekovski in [3] conjectured that each plane graph has an l -facial coloring with at most $3l + 1$ colors. It is known that every plane graph admits a 2-facial, 3-facial, and 4-facial coloring (see [2, 3]) with at most 8, 11, and 15 colors, respectively.

We consider an edge coloring based on the same conditions as the mentioned l -facial vertex coloring.

An l -facial edge coloring, l -FEC, of a plane graph G with k colors is a mapping $\varphi : E(G) \rightarrow \{1, 2, \dots, k\}$, such that for any pair of distinct edges e_1, e_2 of G at distance at most l on a boundary of some face, $\varphi(e_1) \neq \varphi(e_2)$. The minimum number of colors for which G admits an l -FEC is the l -facial chromatic index, $\chi'_{lf}(G)$. We present that there exists 2-FEC for each plane graph G where $\chi'_{lf}(G) \leq 7$.

the need for teaching the concept of average, both as a statistical idea for describing and making sense of data sets and as a computational algorithm for solving problems.

Modern Digital Technologies in the Teaching of Probability

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Modern digital technologies can be appropriately applied in the teaching of probability and statistics. Computer simulations represent an important part of application of digital technologies and they can be used to demonstrate various probability phenomena and patterns, analysis and evaluation of results of random experiments. Well-processed data in the form of summary tables and graphs can help in testing of student's hypotheses or in guiding their considerations and decisions. The goal of the article is to suggest a probabilistic didactic game Chuck and Luck using computer simulation, graphical and tabular view in MS Excel.

Description of Congruence Lattices in CIP Varieties via Priestley Duality

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A Priestley space is a compact, totally order disconnected (CTOD) topological space. There is a duality between the category of Priestley spaces and the category of bounded distributive lattices.

We say that a variety \mathcal{V} of algebras has the Compact Intersection Property (CIP), if the family of compact congruences of every $A \in \mathcal{V}$ is closed under intersection. We provide a description of congruence lattices in several types of a finitely generated, congruence-distributive CIP varieties via Priestley duality.

On Integration with Respect to a Probabilistic Measure

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One of the most important concepts in mathematical analysis is the classical measure and integration theory. However, in practical situations this theory has some natural restrictions due to its additivity property. Also, in real situations we often have only a probabilistic information about measure of a set, therefore a concept of probabilistic (sub)measures was introduced. Especially, we deal with the so-called τ_T - (sub)measures introduced in [2]. They are closely related to the Menger probabilistic metric spaces. In our contribution we provide a concept of a probabilistic integral of real non-negative functions with respect to a distance distribution functions-valued measure γ . For a simple measurable function $f : \Omega \rightarrow \overline{\mathbb{R}}_+$ of the form $f = \sum_{i=1}^n x_i \chi_{E_i}$, $E_i \in \Sigma$, the integral is defined as follows

$$\int_E f d\gamma = \int_E \left(\sum_{i=1}^n x_i \chi_{E_i} \right) d\gamma := \bigoplus_{i=1}^n \gamma_{E \cap E_i} \left(\frac{j}{x_i} \right), \quad E \in \Sigma,$$

where Σ is a ring of subsets of $\Omega \neq \emptyset$ and \bigoplus is an addition in the Menger space. Integral of a non-negative measurable function f may be obtained as the supremum of integrals of all simple functions $f_n \leq f$. We describe some basic properties of this integral. Some results are discussed from the topological point of view. These results are closely related to [1].

Acknowledgement. The present work was supported by the Internal grant VVGS-PF-2012-36.

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Properties of Subgraphs of 4-Critical Planar Graphs

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A graph G is k -critical if $\chi(G) = k$ and $\chi(H) < \chi(G)$ for every proper sub-graph H of G . It is well known that every planar graph is 4-colorable, therefore class of 4-critical planar graphs is extremal class of planar graphs (in sense of colorability). Koester [2] proved that every 4-critical planar graph contains vertex of degree at most 4 and this bound is best possible. We will prove that every 4-critical planar graph contains edge of low weight.

Fabrici and Jendroľ [1] showed that if 3-connected planar graph G contains k -vertex path P_k then it contains P_k such that $\Delta_G(P_k) \leq 5k$. We will prove similar theorem for 4-critical planar graphs and show that bound is, in this case, exponential.

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Motion Word Problems

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In this contribution we characterize motion word problems. We determine their position in the national curriculum and in Slovak textbooks before and after the school reform in 2008. Further, we present some ways to solve these word problems and then show illustrations of pupils' solution.

Students' Conceptual and Procedural Knowledge of Some Statistical Concepts

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Although the most common statistical idea encountered in everyday life and school contexts is the average value of a set of data, studies of students' understanding of average show that students lack the full range of meanings that average is used to convey. This contribution deals with conceptual and procedural understanding. Conceptual understanding of arithmetic mean includes both an understanding of the computational algorithm and the statistical aspects of the concept. In contrast, procedural understanding is made up of rules, procedures or computations necessary to compute arithmetic mean.

We focused on students' understanding of the arithmetic mean by assessing their understanding of the computational algorithm. The results of many researches and also our study showed that the majority of the students know the "add-them-all-up-and-divide" averaging algorithm, but only some of the students are able to correctly apply the algorithm to solve a contextualized average problem. The average concept is more complex than the simplicity suggested by the computational algorithm, but also indicates