Jednota slovenských matematikov a fyzikov
Pobočka Košice

Prírodovedecká fakulta UPJŠ
Ústav matematických vied

Fakulta elektrotechniky a informatiky TU
Katedra matematiky a teoretickej informatiky

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16. Konferencia košických matematikov

Herľany

25. – 28. marca 2015

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Predhovor

Vážení priatelia, milí hostia, kolegyne a kolegovia,

vitajte na 16. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Na tento rok pripadá 25. výročie založenia ZSVTS. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice – v Herľanoch.

Nápad organizovať konferenciu tohto typu vznikol v našej pobočke JSMF pod vedením prof. Jendroľa pred viac ako šestnástimi rokmi. Bola za tým myšlienka, že ľudia profesionálne sa zaoberajúci matematikou v jej rôznych podobách (učitelia, vedci, aplikovaní matematici) a žijúci na východe Slovenska by mali mať možnosť sa pravidelnejšie stretávať, podeliť sa s rovnako „postihnutými“ kolegami o svoje radosti i starosti súvisiace s prácou matematika či matematikára; následne spoločne alebo s ďalšími spriaznenými dušami hľadať riešenia či východiská z problémov. Prípadne si vzájomne pomáhať a povzbudiť sa navzájom. Ďalej to bola predstava, že by malo ísť o serióznu konferenciu s kvalitným obsahom, najmä pozvanými prednáškami. Od začiatku boli na ňu pozývaní prednášajúci s cieľom, aby to boli či už zrelé alebo práve vychádzajúce kvalitné osobnosti, známe vo svojom prostredí, s cieľom dozvedieť sa nové veci, nadviazať nové či upevniť staré kontakty. Viaceré z týchto prednášok mali taký pozitívny ohlas, že ich autori boli pozvaní prednášať aj na iných konferenciách. Tohtoročná konferencia má malé *informatické* jubileum – 2^4 rokov.

To, že Konferencia košických matematikov sa koná po 16. krát je len potvrdením, že tieto myšlienky našli úrodnú pôdu. Každoročne sme na nej mali skvelých prednášajúcich. Na výbere a príprave konferencie sa pracuje celý rok. O výbere pozvaných prednášajúcich sa v podstate rozhoduje na tradičnom každoročnom stretnutí výboru košickej pobočky JSMF s košickými profesormi matematiky a vedúcimi košických matematických pracovísk, vrátane riaditeľa Gymnázia na Poštovej ulici, ktoré má matematické triedy.

Za tých 16 rokov sa vykryštalizovala aj štruktúra konferencie. Prvé dva dni (streda a štvrtok) sú venované najmä mladým začínajúcim matematikom. Mnohí dnes už veľmi úspešní kolegovia mali svoje prvé verejné odborné či vedecké vystúpenie práve na našej konferencii. Vystúpenia mladých kolegov majú z roka na rok vyššiu úroveň, čo organizátorov veľmi teší. V piatok a v sobotu dopoludnia sa konajú najmä pozvané prednášky, aby sa na nich mohlo zúčastniť čo najviac účastníkov. Spoločenský piatkový večer je organizovaný tak, aby bolo možné v menších skupinách pri poháriku vínka predebatovať rôzne otázky.

Aj tento rok sa nám podarilo získať viacero výrazných osobností. Pozvanie prednášať prijali: doc. Dr. P. Faliszewski, PhD. (KI FEAIE AGHU Kraków), doc. RNDr. P. Frolkovič, PhD. (KMaDG SvF STU Bratislava), prof. Ing. V. Gazda, PhD. (KF EkF TU Košice), Dr. Z. Chladná, (KAMaŠ FMFI UK Bratislava), Dr. R. Kalinowski (FMS KMD AGHU Kraków), Mgr. J. Kiseľák, PhD. (ÚMV PF UPJŠ Košice), doc. RNDr. V. Pirč, CSc. (KMTI FEI TU Košice), RNDr. R. Plch, Ph.D. (ÚMaS PF MU Brno) a doc. PaedDr. K. Žilková (KPaEM PF KU Ružomberok).

Prajeme vám príjemný pobyt v Herľanoch

Organizačný výbor: Ján Buša
Jozef Doboš
Róbert Hajduk

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Invited lectures

Challenges in Modelling Vaccination Behavior

Zuzana Chladná

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Slovakia is one of the few European countries, where mandatory vaccination is enacted by law. Thanks to mandatory vaccination, incidence of infectious diseases in Slovakia is quite low compared to other countries. However, recent frequent mass media discussions about the adverse effects of vaccination have gradually decreased compliance of parents; therefore, a shift from mandatory vaccination to a voluntary scheme has become an issue.

This work is an attempt to respond to this situation. Our aim is to predict the epidemiological situation after potential abolition of the mandatory vaccination. For our modelling purposes we adopt the standard susceptible-infected-recovered (SIR) model with demographic effects and vaccination (see e.g. [1]).

First, we examine effects of reduced vaccination coverage on epidemiological situation. The importance of spatial heterogeneity of vaccination coverage is explored.

Further, we present a modified SIR model with endogenously modelled vaccination coverage. We assume that this parameter is a result of the so called *vaccination game*. We introduce two approaches how to formally describe incentive to vaccinate. In the first approach an equilibrium vaccination coverage is determined by a game theory concept, while in the second one the resulting equilibrium coverage is determined as the steady-state solution of a system of differential equations. Both approaches are motivated by results of Bauch (2004, 2005). We make a synthesis of both approaches and discuss the results in the context of Slovakia.

Acknowledgement. This study was supported by Slovak Grant Agency APVV-0096-12.

References

[1] Anderson, R. M. – May, R. M.: *Infectious Diseases of Humans: Dynamics and Control*, Oxford University Press, Oxford. (1991).

[2] Bauch, C. T. – Earn, D. J. D.: *Vaccination and the theory of games*, Proceedings of the National Academy of Sciences, (2004), 13391–13394.

[3] Bauch, C. T.: *Dynamic games with imitation dynamic predict vaccinating behavior*, Proceedings of the Royal Society of London, Series B, 272(1573), (2005), 1669–1675.

[4] Hofbauer, J. – Sigmund, K.: *Evolutionary game dynamics*, Bull. Am. Math. Soc., 40, (2003), 479–519.

How to Elect a Committee of Representatives and Live to See the Result

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In this talk we will discuss a mathematical model of elections, where the goal is to elect some committee of representatives (for example, the members of the parliament). Briefly put, the voters rank all the possible candidates from the most desriable one to the least desirable one, and then — using some voting rule — we need to pick the k committee members (where k is a parameter of the procedure). We discuss a number of axiomatic properties that such committee-selection rules should have, and discuss a number of rules. It turns out that one of the most appealing committee selection rules is computationally difficult to compute (NP-hard).

We will discuss this notion and explain how one can circumvent this hardness result.

Advection Equation in Education, Research and Applications

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The advection equation in its simplest form describes a passive transport of a substance in a given flow. More complicated forms of similar processes are modeled by nonlinear advection equations when it is very often coupled to other processes like diffusion and reactions.

The advection equation can be found in many areas of mathematics including education, research and several applications. In this talk we try to cover in a popular form our experiences of dealing with the advection equation and its numerical solution in all three areas.

In education the advection equation can be found especially in lessons on partial differential equations and in the lectures on fluid dynamics. In fact, the linear advection equation with constant coefficients can be seen as the simplest example of partial differential equation. Teaching this topic can bring students a lot of experiences in an attractive form.

The advection equation is a main focus of many research publications not only for standard areas like modelling flow and transport problems, but also in nonstandard topics like optic flow or image processing. It is then a natural consequence that the advection equation and its numerical solution is used in many real life applications.

In this talk we briefly mention some of applications of advection like the flow and transport in porous media or the so called level set methods to model moving interfaces e.g. in two-phase flows problems, the segmentation of biomedical data or forest fire spreading.

On an Equilibrium Price in a Complex Local Structure of the Buyers and Sellers

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The classical view on the price determination is based on the perfect competition assumptions and, in this way, offers the idea of the unique price in the market. Empirical research often contradicts this view and economists explain this fact by market imperfections. The alternative continual model, based on the price-location competition among the retailers has been presented in the seminal work of Hotelling (1929).

A major novelty of our work is the tight integration of an equilibrium price model and weighted digraph analysis. We are using geodesic distances between nodes in a graph representation of the urban area respecting its real topology and finiteness of the set of the customer locations. The model focuses on the complex relations between consumers and retailers, which are described by the means of digraphs affording computational tractability.

Keywords: Graph, Demand, Min-Cost Flow, Nash Equilibrium

On Some Partitions and Decompositions of Graphs

Rafał Kalinowski

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Partitions and decompositions of graphs constitute an important and wide area in graph theory. A *partition of a graph* $G = (V, E)$ is a partition of the vertex set V, and a *decomposition of a graph* is a partition of the edge set E. In the talk, several problems will be discussed.

A graph G of order n is called *arbitrarily partitionable* (AP for short) if, for every sequence $(n_1, \ldots, n_k)$ of positive integers with $n_1 + \cdots + n_k = n$, there exists a partition $(V_1, \dots, V_k)$ of the vertex set $V(G)$ such that V_i induces a connected subgraph of order n_i for $i = 1, \dots, k$. A *sun with r rays* is a graph of order $n \geq 2r$ with r pendant vertices $u_1, \dots, u_r$ whose deletion yields a cycle C_{n-r}, and each vertex v_i on C_{n-r} adjacent to u_i is of degree three. We characterize all AP suns with at most three rays. We also show that every connected graph G of order $n \geq 22$ and with $\|G\| > \binom{n-4}{2} + 12$ edges is AP or belongs to few classes of exceptional graphs. Moreover, a conjecture that the Cartesian product of AP graphs is AP will be discussed.

Another problem concerns decompositions of a transitive tournament TT_n into oriented graphs of size less than five. This problem was motivated by the following theorem of Sali and Simonyi: every self-complementary graph has an orientation that decomposes TT_n.

We also introduce an *edge-distinguishing index* of a graph G as the minimum number of colours in a proper edge colouring of G such that each edge has a distinct coloured closed neighbourhood. We determine this invariant for paths, cycles and complete graphs.

The last topic concerns the palette index of a graph. A proper colouring of edges of a graph defines for each vertex v the set $S(v)$ of colours of edges incident to v, called the *palette* of a vertex v. The *palette index* of G is the minimum number of distinct palettes in G taken over all possible proper edge colourings of a graph. We determine the palette index of complete graphs and of cubic graphs.

On Integral Equations and Related Problems

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For several years the subject of functional equations has held a considerable place in the attention of mathematicians. Of late years this attention has been oriented to a particular kind of functional equation, an integral equation. Among other reasons, because a variety of applied problems have their natural mathematical setting as integral equation [1]. Four main types of nonlinear one-dimensional integral equations can be written as

$$\delta\, y(t) = \lambda\, f(x) + \int_a^{\odot} K(x, y(t), t)\mathrm{d}t, \tag{1}$$

where $f \in C([a, b], \mathbb{R})$, $K \in C([a, b]^3, \mathbb{R})$, $\odot$ is either x or b, y is unknown function and λ, a, b, δ are real constants. It is also well known, that a large class of initial and boundary value problems, associated with differential equation, can be reduced to integral equations of the form (1). In the special case, when δ equals zero, these equations are referred to as equations of the first kind. Even in the linear case they need special discussion, although it is not at first sight clear why.

We would like to point out some of the remarkable peculiarities of an essential nature in solvability theory [3] and give some available numerical methods for their solutions [2]. In the end we mention a specific type of integral equation, which involves function composition of unknown variable. We also underline issues closely related to the numerical solutions.

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References

[1] Jerri, A. J.: *Introduction to integral equations with applications.* 2nd edn. Wiley-Interscience, New York (1999)

[2] Mikhlin, S. G. – Smolitski, Kh. L.: *Approximate methods for solution of differential and integral equations.* American Elsevier, New York (1967)

[3] Tricomi, F. G.: *Integral equations*, Interscience Publishers, New York: Dover (1957)

Mathematics for Blind Students

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The contribution describes using of some mathematical learning methods for the visually impared students. The study of Mathematics is one of the most difficult subject at the Technical University of Košice. The problem is so complex that disable access to scientific education for a generation of blind or severely visually impaired students.

The main goals of the Access Centre of the Technical University of Košice is to improve:

- The current situation of the blind in the work with mathematical and technical text for the study of technical fields.
- Alternatives to the use of Braille in the work with mathematical text.
- The possibilities of mathematical text writing for programs with voice output (screen readers).
- Cooperation between the visually impared students and their teachers.
- Software for the blind to work with mathematical text: Editor Lambda and its applications.
- Opportunities in the field of disclosure of mathematics.

This paper describes in detail characteristics and main features of the Lambda mathematics editor (LAMBDA Linear Access to Mathematics for Braille Devices and Audio-synthesis), with which visually impared students and professionals can write, read and work fairly complex symbolic and mathematical expressions. Individual mathematical symbols, operators, functions is in Lambda editor [1] inserted (written) by user friendly menu, without requiring knowledge of Braille. Each written mathematical expression can be read by screen reader or can be visually displayed by traditional graphical method and also printed in Braille There are described also some applications of a mathematical software for the easy access to solution specific tasks for blind students.

References

[1] *Mathematical Editor Lambda.* Lambda editor is available at http://www.teiresias.muni.cz/czbraille8/?strana=download

Interactive Teaching Materials in PDF Format

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The aim of this paper is to present the opportunities provided by the TEX typesetting system and its packages for the creation of interactive teaching materials in PDF format. On the examples of learning objects from mathematics we show the use and possibilities of interactive 3D and 2D graphics, animations, games, quizzes and tests. We introduce tools for creating these objects and benefits for their creators and users. These include high-quality mathematical typography, platform independence, easy accessibility (only free Adobe Reader is needed), evaluation of tests without a Internet connection, use for interactive whiteboards and much more. So we can make the learning process more attractive and more enjoyable for students (with minimum costs).

References

[1] Grahn, A.: *The animate Package.* CTAN [online]. 2011 [cit. 2012-04-26]. Available from: http://www.ctan.org/tex-archive/macros/latex/contrib/animate.

[2] Grahn, A.: *The media9 package.* CTAN [online]. 2012 [cit. 2012-04-26]. Available from: `http://www.ctan.org/pkg/media9`.

[3] Mařík, R.: *The jeopardy package.* CTAN [online]. 2010 [cit. 2011-01-10]. Available from: `http://www.ctan.org/tex-archive/help/Catalogue/entries/jeopardy.html`.

[4] Mařík, R. – Plch, R. – Šarmanová, P.: *Tvorba interaktivních testů pomocí systému AcroTEX.* Zpravodaj Československého sdružení uživatelů TEXu. 2010, vol. 20, no. 4, s. 266–291, ISSN 1211-6661 (tištěná verze), ISSN 1213-8185 (online verze).

[5] Plch, R. – Šarmanová, P.: *Interaktivní 3D grafika v HTML a PDF dokumentech.* Zpravodaj Československého sdružení uživatelů TEXu. 2008, roč. 18, 1–2, s. 76–92. ISSN 1211-6661. Available from: `http://bulletin.cstug.cz/pdf/bul_0812.pdf`.

[6] Plch, R. – Šarmanová, P. – Sojka, P.: *Integrální počet funkcí více proměnných* [online]. 1 vyd. Brno: Masarykova univerzita, 2009 [cit. 2012-04-27]. Elportál. Available from: `http://is.muni.cz/elportal/?id=845824`.

[7] Plch, R. – Šarmanová, P.: *Interaktivní hry a testy k procvičení integrálního počtu funkcí více proměnných.* 7. Konference o matematice a fyzice na vysokých školách technických. Brno : Univerzita obrany, 2011. od s. 325–332, 8 s. ISBN 978-80-7231-815-5.

[8] Plch, R.: *Interaktivní 3D grafika v PDF dokumentech.* Zpravodaj Československého sdružení uživatelů TEXu. 2013, vol. 23, issue 1, s. 31–43. DOI: 10.5300/2013-1/31. Available from: `http://bulletin.cstug.cz/doi.php/10.5300/2013-1/31`.

[9] Story, D. P.: *Rich media annotations and AcroFLeX.* TUGboat, Volume 30 (2009), No. 2. Available from: `https://www.tug.org/TUGboat/tb30-2/tb95story.pdf`.

Conceptions and Misconceptions about Geometric Shapes

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Thorough analysis of the learning process in mathematics is important determinant of the quality improvement in mathematical education. Levels of cognitive process and geometric thinking of pupils are the aims of researches of many foreign didactic mathematicians. Their researches specialize in different levels of education (e.g. children in preschool age: Levenson, Tirosh and Tsamir, 2011; pupils in secondary school: Usiskin 1982; pre-service mathematics teachers: Gontay and Paksu, 2012; Erdogan and Dur, 2014). Theory of geometric thinking is the theoretical background of the studies mentioned above. Dutch math teachers, Dina van Hiele-Geldof and Pierre van Hiele, are the authors of the theory (1957). Their model of cognitive process in geometry consists of two parts:

1. Levels of thinking in geometry and their characteristics;
2. Phases of learning, which is devoted to teachers: how to effectively organize the teaching of geometry.

The levels of geometric thinking are called: Visualization, Analysis, Abstraction, Deduction and Rigor. The aim of the lecture is:

- Explain the theoretical principles of the thinking levels in geometry;
- Adduce examples how preschool children recognize different types of geometric shapes;
- Show results of our own research about quadrilateral conceptions and misconceptions in pre-service teachers;
- Identify common characteristics and also differences in outcomes of foreign and domestic researches, especially in terms of differences in mathematical terminology.

Identifying the pupil's geometric levels of thinking allows us to diagnose their misconceptions and design an individual model of learning in order to correct the misconceptions.

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References

[1] Erdogan, E. O. – Dur, Z.: *Preservice Mathematics Teachers' Personal Figural Concepts and Classifications About Quadrilaterals.* Australian Journal of Teacher Education, vol. 39, issue 6, 2014, pp. 106–133.

[2] Gontay, E. – Duatepe Paksu, A.: *Preservice Mathematics Teachers' Understandings of The Class Inclusion Between Kite and Square.* In Procedia – Social and Behavioral Sciences 55, 2012, vol. 55, pp. 782–788.

[3] Levenson, E. – Tirosh, D. – Tsamir, P.: *Preschool Geometry. Theory, Research and Practical Perspectives.* Rotterdam: Sense Publishers. 2011. ISBN 978-94-6091-600-7 (e-book)

[4] Usiskin, Z.: *Van Hiele levels and achievement in secondary school geometry.* Chicago Univ., National Inst. of Education. 1982. Available at: `http://psych.stanford.edu/~jlm/pdfs/Usiskin82AssessingvanHiele.pdf`

Conference contributions

Scale Invariance of the Equivalent Utility Principle under Cumulative Prospect Theory

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Recently, the monotone, not necessarily additive set functions and the integrals with respect to such functions are applied in various branches of economics, e.g. in decision making, finance and insurance (see a survey paper by Heilpern (2002)). In the talk we present an application of the *Choquet integral* in the theory of premium principles. Let us recall that given a *distortion function* $g : [0,1] \to [0,1]$, that is an increasing function with $g(0) = 0$ and $g(1) = 1$, the Choquet integral related to g is given by

$$E_g(X) = \int_{-\infty}^{0} (g(P(X > t)) - 1)\, dt + \int_{0}^{\infty} g(P(X > t))\, dt.$$

Consider an insurance company having the initial wealth w and a utility function u. The company covers a risk treated as a non-negative random variable. Roughly speaking, a premium principle is a rule for assigning a premium to an insurance risk. One of the frequently applied methods of pricing insurance contracts is the *Principle of Equivalent Utility*. Under the Cumulative Prospect Theory a premium principle $H(X)$ for risk X is a solution of the equation

$$u(w) = E_{gh}[u(w + H(X) - X)],$$

where

$$E_{gh}(X) = E_g(\max\{X, 0\}) - E_h(\max\{-X, 0\})$$

is the generalized Choquet integral related to the distortion functions g (for gains) and h (for losses). In a recent paper by Kałuszka and Krzeszowiec (2012) several properties of the premium have been considered. One of them is a scale invariance, known also as a positive homogeneity. Let us recall that a premium principle is said to be *scale invariant* provided $H(aX) = aH(X)$ for all feasible risks X and $a > 0$. In this talk we show that a scale invariance of a premium principle just for two particular values of parameter a implies its scale invariance.

References

[1] Heilpern, S.: *Using Choquet integral in economics.* Statistical Papers, 43 (2002), 53–73.

[2] Kałuszka, M. – Krzeszowiec, M.: *Pricing insurance contracts under Cumulative Prospect Theory.* Insurance: Mathematics and Economics 50 (2012), 159–166.

Ideals on the Real Line

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Except for the well-known ideals such as ideal of measure zero sets and ideal of meager sets, there are also other lesser-known ideals on the real line such as ideal of bounded sets, ideal of not dominating sets, etc. In the presented talk we introduce these ideals and compare some of their properties.

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Fourier Series as Multi-Stimuli and Operation with Them

Tomáš Gregor

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Fourier series is one of the basic tools dealing with waves. It arises as a solution of the wave equation. Fourier series on the interval $(-\pi, \pi)$ is a sum of sine and cosine function

$$\sum_{k=0}^{\infty} a_k \cos(kx) + b_k \sin(kx),$$

where a_k, b_k are constants. A new view of Fourier series provides the theory of multi-stimuli (multi-polarity). The idea of multi-stimuli was described in the paper [1]. Here, the complex plane is constructed as k-stimuli space over the semi-field of all non-negative real numbers. According to this paper, we decompose the k-th term of Fourier series into the $(k+1)$-stimuli space. Each of the multi-stimuli coordinates is non-negative. Due to the cancellation law, there are just two non-zero k-stimuli coordinates. We also deal with operations with Fourier series in this form. We investigate the operation of addition given coordinate-wisely, and the operation of multiplication given by convolution.

Acknowledgement. This work was supported by grant VEGA 2/0178/14 and by Project of SAS and NASU "Vector valued measures and integration in polarized vector spaces".

References

[1] Gregor, T. – Haluška, J.: *Lexicographical ordering and field operations in the complex plane.* Matematychni Studii 41(2) (2014), 123–133.

Integration with Respect to Probabilistic-Valued Measures

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The aim of this contribution is to present a Lebesque-type approach to the integration of non-negative real-valued functions with respect to probabilistic-valued decomposable measures [1]. They are set functions taking values in the set Δ^+ of distance distribution functions of non-negative random variables. We will discuss probablistic-valued integral as an efficient tool for dealing with certain types of imprecise information.

Acknowledgement. The present work was supported by the Internal grant VVGS-2014-182 and VVGS-PF-2014-453.

References

[1] Halčinová, L. – Hutník, O.: *An integral with respect to probabilistic-valued decomposable measures,* International Journal of Approximate Reasoning 55 (2014), 1469–1484.

Oscillation Criteria for Fourth Order Trinomial Delay Differential Equations

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Linear differential equations of fourth order form part of an immense collection of higher-order differential equations and are encountered in various fields of science and engineering as the more basic mathematical models.

For instance, it is well known that the problem of beam deflection in linear theory of elasticity is represented by the classical linear fourth order equation

$$y^{(4)}(t) + q(t)y(t) = 0,$$

where $y(t)$ approximates the shape of a beam, deflected from the equilibrium due to some external forces.

Following [1], let the motion of a (sufficiently) long beam be governed by the following delay differential equation

$$y^{(4)}(t) + p(t)y'(t) + q(t)y(\tau(t)) = 0, \quad \tau(t) \le t, \tag{E}$$

where the middle term is incorporated to control the slope of the beam. As the beam undergoes horizontal oscillations, the studied motion is described more accurately due to presence of delay.

The primary motivations behind this talk are twofold. First, a continuation of the pioneering work in [1], where some sufficient conditions ensuring that any solution of (E) either oscillates or converges to zero have been established. Secondly, there was a thought of a missing analogy with the investigation of trinomial differential equations where the derivative order of the first and the middle term differs by two. In such case, the examination of studied equations can be greatly simplified using their equivalent binomial representation [2].

A new approach discussed in the talk uses a positive solution of an auxiliary third and second order differential equation to obtain an associated binomial form of (E). Contrary to known results, all nonoscillatory solutions are eliminated by comparison with a suitable couple of first-order delay differential equations. In view of the mentioned beam deflection problem, this ensures positive as well as negative displacements of the beam.

References

[1] Hou, Ch. – Chengmin, S. S.: *Asymptotic dichotomy in a class of fourth-order nonlinear delay differential equations with damping.* Abstract and Applied Analysis, 2009, 1–9.

[2] Baculíková, B. – Džurina, J.: *Comparison theorems for the third-order delay trinomial differential equations.* Advances in Difference Equations, 2010 (2010), 1–9.

Centralizers of Monounary Algebras and Green's Relations

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For a given (partial) algebra $\mathcal{A}$, its (first) centralizer is defined as the set of those mappings of $\mathcal{A}$ into $\mathcal{A}$, which commute with all basic operations of $\mathcal{A}$. The second centralizer is the set of all mappings which commute with all elements of the first centralizer. The results concern monounary algebras with the same first and second centralizer. We characterize Green's relations on the semigroup $(C, \circ)$, where C is the centralizer with the above property. The Green's relation $\mathcal{R}$ is an equivalence on C corresponding to the quasiorder $\leq_{\mathcal{R}}$; the quasiorder $\leq_{\mathcal{R}}$ is described as well.

Fragmentation in Mathematics Education

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Fragmentation is a feature that can be observed in most of contemporary human activities. The interruptions of a continuous information flow are present almost everywhere – in media, work processes and even leisure. On examples from media we present proofs of such fragmentation. Naturally it has consequences also in educational activities. One of such consequences is a tendency of students to multitasking, by what we mean performing different activities simultaneously. We present effects of multitasking in mathematical education and stress its specificity in comparison to more data-oriented subjects.

Maximum Colouring

Stanislav Jendroľ and Michaela Vrbjarová

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A k-edge-coloring of G having a property that for every vertex v of degree $d_G(v) = d$, $d \geq r$, the maximum color, that is present at vertex v, occurs at v exactly r times, is called an r-maximum k-edge-coloring of G. The r-maximum index $\chi'_r(G)$ is defined to be the minimum number k of colors needed for an r-maximum k-edge-coloring of graph G. We show that $\chi'_r(G) \leq 3$ for any nontrivial connected graph G and $r = 1$ or 2. The bound 3 is tight. We characterize all graphs G with $\chi'_1(G) = i, i = 1, 2, 3$ and determine the precise value of the r-maximum index, $r \geq 1$, for trees and complete graphs.

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Structure of 3-Paths in Graphs with Bounded Maximum Average Degree

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We study the existence of paths on 3 vertices with given degree sequence in sparse graphs. A path of type (x, y, z) $(x, y, z \in \mathbb{N}^*)$ is a path uvw such that the degree of u (resp. v, w) is at most x (resp. y, z). The *maximum average degree* of a graph G, denoted by $\mathrm{mad}(G)$, is defined as the maximum of the average degrees $\mathrm{ad}(H) = \frac{2|E(H)|}{|V(H)|}$ taken over all the subgraphs H of G. We prove that every graph with minimum degree at least 2 and maximum average degree strictly less than m has a path of one of the types

1. $(2,\infty,2)$, $(2,8,3)$, $(4,3,5)$, $(5,2,5)$ if $m=\frac{15}{4}$;
2. $(2,\infty,2)$, $(2,5,3)$, $(3,2,4)$, $(3,3,3)$ if $m=\frac{10}{3}$;
3. $(2,2,\infty)$, $(2,3,4)$, $(2,5,2)$ if $m=3$;
4. $(2,2,13)$, $(2,3,3)$, $(2,4,2)$ if $m=\frac{14}{5}$;
5. $(2,2,i)$, $(2,3,2)$ if $m=\frac{3(i+1)}{i+2}$ for $4\leq i\leq 7$;
6. $(2,2,3)$ if $m=\frac{12}{5}$;
7. $(2,2,2)$ if $m=\frac{9}{4}$.

Moreover, the optimality of these results is discussed.

MLE and QLS Method for Connection between Uniform and Serial Correlation Structure

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The object of the study is Growth curve model, which represents connection between regression analysis and analysis of variance. Standard Growth curve model is of the form: $Y = XBZ + \epsilon$, where ϵ is matrix of random errors which has normal distribution, mean of ϵ is zero and variance of vector $\mathrm{vec}\epsilon$ is of the form $\Sigma\otimes I$. We study one of possible special structure of Σ, which has only 2 parameters. The estimation is carried out using the quasi least squares method and maximum likelihood estimator. Theoretical knowledge is supplemented by real data example.

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Separation Properties in Topological Inverse Semigroups

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It is well-known that in topological groups the separation axioms T_0 and T_2 are equivalent. This equivalence disappears if we consider more general algebraic, structures topological inverse semigroups, or weaken the connection between the topological and the algebraic structures, semitopological groups, inverse semigroups.

A. Conte gave sufficient conditions for topological inverse semigroups which ensure the validity of the separation axioms T_0, T_1, T_2 [1], and those falling between T_0 and T_1 [2]. He also gave examples of topological inverse semigroups where the mentioned separation axioms are not equivalent. His idea was to require separation-like conditions related to the set of idempotents, or in the relation between an idempotent and an other element. Recent results underline the importamnce of the study of the separation axioms in such structures e.g. [3], [4] and [5].

The aim of this presentation is to study in semitopological and topological inverse semigroups separation axioms between T_1 and T_2, T_3 and also those which satisfy certain order conditions.

References

[1] Conte, A.: *Assiomi di separazione nei gruppidi topologici.* Richerche di Matematica 17 (1969), 168–180.

[2] Conte, A.: *Gruppidi topologici soddisfacenti ad assiomi de separationi intermedi fra T_0 e T_1.* Atti. Accad. Sci. Torino Cl. Sci. Fis. Mat. Nat. 5 (1971), 185–193.

[3] Avila, J. A.: *Spec(R) y axiomas de separacion entre T_0 y T_1.* Divulg. Mat. 13 (2) (2005), 90–98.

[4] Ge, X. – Yang, S. – Cao, Y.: *Remarks on S_i-separation axioms.* Filomat 26:4 (2012). 839–844 `DOI 10.2298/FIL1204839G`.

[5] Tkachenko, M.: *Axioms of separation in semitopological groups and related functors.* Topology and its Applications, 162 (2014), 364–376.

On Some Nontransitive Relation

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The talk is about relation defined in a set of so-called *series of heads and tails* (of established length) occurring in context of the random experiment carried out in Penney-ante game. It seems that this relation is transitive – such a conclusion seems to be obvious. It is shown that irrespectively of the length of heads and tails series, the discussed relation is not a transitive relation.

References

[1] Guibas, L. J. – Odlyzko, A. M.: *String overlaps, pattern matching, and nontransitive games*, Journal of Combinatorial Theory, **30** (1981), seria A, 183–208.

[2] Krech, I.: *Osobliwe wlasnosci modeli probabilistycznych czekania na serie sukcesow i porazek*, Ann. Acad. Paed. Cracov. 5 Studia Ad Calculum Probabilistis Eiusque Didacticam Pertinentia 1 (2002), 39–55.

[3] Krech, I.: *Waiting for series of colours and properties of some relations in a set of these series*, Acta Univ. Purkynianae 72 Studia Mathematica (2001), 124–128.

[4] Penney, W.: *Problem 95: Penney-Ante*, Jurnal of Recreational Mathematics, **7** (1974), 321.

[5] Plocki, A.: *Pravdepodobnosť okolo nás*, Katolícka univerzita, Ružomberok 2007.

Spectral Functions Density of Toeplitz Localization Operators

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We study the density of the set of spectral functions corresponding to Toeplitz localization operators (TLO's) based on the Calderón and Gabor reproducing formula in a unified way as introduced in [2]. Because we deal with TLO's symbols depending on the first coordinate in the phase space, the spectral functions can be written as convolution of the TLO's symbol with a kernel function involving admissible wavelet/window. From this moment we may use the Wiener's deconvolution technique on the real line to describe the density of spectral functions. Indeed, under assumption that the Fourier transform of kernel function does not vanish on the real line we prove that the set of spectral functions is dense in the C^*-algebra of uniformly continuous functions on the real line. The result of this general kind is then applied to the important special case of a parametric family of wavelets related to Laguerre functions studied in [1]. Indeed, we prove the conjecture about density of spectral functions for Toeplitz operators acting on poly-analytic Bergman spaces on the upper half-plane from [3].

Acknowledgement

The support of the grant VEGA 1/0345/15, VVGS-2014-182 and VVGS-PF-2014-453 is kindly announced.

References

[1] Hutník, O.: *Wavelets from Laguerre polynomials and Toeplitz-type operators.* Integr. Equ. Oper. Theory **71**(3) (2011) 357–388.

[2] Hutník, O. – Hutníková, M.: *On Toeplitz localization operators.* Arch. Math. **97**(4) (2011) 333–344.

[3] Hutník, O. – Hutníková, M.: *Toeplitz operators on poly-analytic spaces via time-scale analysis.* Oper. Matrices **8**(4) (2014) 1107–1129.

Blended Learning In Mathematics Course

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Integration of Information and Communication Technologies (ICT) into the education and learning process resulted in electronic learning or e-learning. E-learning is a modern concept and an alternative to traditional forms of education (face-to-face education). E-learning has a lot of advantages, but also disadvantages. On one side e-learning is more effective and does not have time and location restrictions. On the other side, however, e-learning is missing the interaction among students and teachers [[4, 5]]. Therefore new concepts combine both traditional and online learning. This combination is also called blended or hybrid learning [[6]]. Blended learning consists of two basic parts: face-to-face learning based on classroom teaching and online learning.

This contribution deals with application of blended learning to the basic undergraduate course Mathematics 2 at Faculty of Mining, Ecology, Process Control and Geotechnology, Technical University of Košice. This course is oriented on analytical geometry and infinitesimal calculus. During the first phase of blended learning implementation we focused on creating and introducing the most important missing electronic materials – online collection of solved examples – into the education process. In our contribution we will present this online collection which was created in blogging system Blogger. Blogger is a free Google service for creating blogs and supports MathJax for displaying mathematics [[1]]. MathJax is an open source JavaScript display engine for mathematical notations (LaTeX, MathML, AsciiMath) in web pages [[3]]. The online collection of solved examples was created in 2014 for both full-time and distance students of our faculty. In conclusion, we will present the results and experiences with introducing electronic materials into education process.

Acknowledgement. The present work was supported by Slovak Research and Development Agency under the contracts No. APVV-0482-11, VEGA grants No. 1/0529/15, No. 1/0908/15 and KEGA grant 040TUKE-4/2014.

References

[1] *Blogger.* [Online]. Retrieved March 12, 2015 from `https://blogger.com/`

[2] *Matematika 2* [Online] Retrieved March 12, 2015 from `http://riesenepriklady2.blogspot.sk`

[3] *MathJax.* [Online]. Retrieved March 12, 2015 from `http://www.mathjax.org/`

[4] Mobbs, R.: *Disadvantages of eLearning* [Online]. Retrieved March 12, 2015 from `http://www.le.ac.uk/users/rjm1/etutor/elearning/disadvofelearning.html`

[5] Kruse, K.: *The Benefits and Drawbacks of e-Learning.* [Online]. Retrieved March 12, 2015 from `http://www.itsmcampus.com/downloads/The_Benefits_and_Drawbacks_of_e.doc`

[6] Osguthorpe, R. T. – Graham, C. R.: *Blended learning environments: Definitions and directions.* The Quarterly Review of Distance Education, 4(3) (2003).

The Geometric Interpretation of Solutions of the Inequalities and Their Systems with the Help of GeoGebra

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Modern trends in the teaching of almost all subjects are directed to the use of information and communication technologies. Math is doing well. GeoGebra is one of the most commonly used dynamic geometry systems in teaching of mathematics. Almost all the suggestions for the use of dynamic geometry systems available on the Internet and in the literature are devoted only to the geometry. Recently, there are already examples of the use of GeoGebra not in geometry. Also, this post shows another use of this program, in solving word problems leading to tackling inequalities and their systems and also by the creation of graphical models to solve a optimization problems.

Acknowledgement. The support of the grant APVV-0715-12 is kindly announced.

Modelling Practical Placement of Trainee Teachers to Schools

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At Pavol Jozef Šafárik University teachers-to-be students specialize in two subjects. During their studies they have to take four practical placements.

In the first two placements, the subjects are performed independently. In the case when one student is assigned to one teacher, a feasible assignment can be found efficiently, by employing network flow techniques. The special rules of our university require, that each teacher is assigned from four to six students. In that case the problem becomes more complicated and it is still open, whether there exists an efficient algorithm for finding a feasible assignment.

In the second two placements both subjects are performed in the same school. This leads to intractable problems even under several strict restrictions concerning the total number of subjects, the number of students each school can accept for practical placement in each subject and the number of acceptable schools each trainee teacher is allowed to list. Due to this intractability results we report on an integer programming model for solving the teacher assignment problem and the results of its application to real data.

Acknowledgement
This work was supported by VVGS-PF-2014-463 (Oceľáková).

References

[1] Cechlárová, K. – Fleiner, T. – Manlove, D. – McBride, I. – Potpinková, E.: *Modelling practical placement of trainee teachers to schools.* CEJOR Central European Journal of Operations Research, 2014, DOI 10.1007/s10100-014-0356-5.

High Performance Computing and Primes

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High-performance computing (HPC) is an information technology which handles large volumes of data and/or a high numerical range. Sometimes high-performance computing is marked as supercomputing, scientific computing or exascale computing. At present HPC is a fundamental innovation technology in the world.

The contribution contains information about our internal project Primes and Supercomputers. The project was solved in collaboration with colleagues from SAS and CERN. One of our key objectives was to develop a common measurement system for application computing power of computers, HPC clusters and supercomputers. The research result is a classification of computing power that allows (and will allow in the future) to compare the computing power of computers, HPC clusters and supercomputers. Our methods, to measure the application computing performance of computers, are based on the use of large Mersenne primes and computations were implemented on SIVVP HPC clusters.

Keywords: *high-performance computing, application computing power, computing power measurment, Mersenne primes*

Acknowledgement. The present work was supported by projects: ITMS 26210120002, 26230120002 and APVV-0526-11.

References

[1] Eukleides: *Stoicheia (Elements), Vol. IX.*, 250 BC.

[2] *Great Internet Mersenne Prime Search.* online on `http://www.mersenne.org/primes/` [cit. 2015-03-10].

[3] *Intel Math Kernel Library Cookbook.* online on `https://software.intel.com/en-us/mkl_cookbook/` [cit. 2015-03-10].

[4] *Slovak Infrastructure for High Performance Computing (SIHPC – SIVVP).* online on `http://www.sivvp.sk/home/` [cit. 2015-03-10].

New Criteria for Inscribability and Non-Inscribability of Polyhedra

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Steinitz's discovery of combinatorial types of polyhedra excluding inscription in a sphere made it meaningful to research this phenomenon. Subsequently, Steinitz's criterion was ramified, exploited and generalized in several directions. Also, it was employed as an inscribability argument for a construction of bipartite inscribable types.

Research line focused on the inscribability property was conducted by Dillencourt via plane representation of inscriptions. By slightly modificating them, he succeeded to prove basic results on inscribability. A crucial shift was done by Andreev's and Rivin's fundamental research of inscribabilty, completed by a characterization of inscribable types of polyhedra via weighting their edges. Again an intimate interconnections between the inscribability and hamiltonicity of polyhedral graphs appeared. Dillencourt formulated and proved a criterion of inscribability in terms of 1-hamiltonicity, a property akin to the hamiltonicity property.

Proceeding in the two lines of reasearch featured, new more sophisticated criteria were obtained to provide with proper argument for inscribability (decomposition hamiltonicity and backbone-cycle hamiltonicity), or non-inscribability (further generalization of structural criteria in a series beginnig with original Steintz' criterion). New criteria make evaluating of inscribability, or non-inscribability, more effective in various contexts.

Light Graphs in Planar Graphs of Large Girth

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A graph H is defined to be light in a graph family $\mathcal{G}$ if there exist finite numbers $\varphi(H, \mathcal{G})$ and $w(H, \mathcal{G})$ such that each $G \in \mathcal{G}$ which contains H as a subgraph, also contains its isomorphic copy K with $\Delta_G(K) \leq \varphi(H, \mathcal{G})$ and $\sum_{x \in V(K)} \deg_G(x) \leq w(H, \mathcal{G})$. In this contribution, we analyze light graphs in families of plane graphs of minimum degree 2 with prescribed girth and no adjacent 2-vertices, specifying several necessary conditions for their lightness and providing sharp bounds on φ and w for light $K_{1,3}$ and C_{10}.

Acknowledgement. The present work was supported by the Internal grant VVGS-PF-2014-447.

On the Problem in the Class of 1-Planar Bipartite Graphs

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A graph $G = (V, E)$ is called 1-planar if it admits a drawing in the plane such that each edge is crossed at most once. We study bipartite 1-planar graphs with prescribed numbers of vertices in partite sets. Bipartite 1-planar graphs are known to have at most $3n - 8$ edges, where n denotes the order of a graph. We show that maximal-size bipartite 1-planar graphs which are almost balanced have not significantly fewer edges than indicated by this upper bound, while the same is not true for unbalanced ones. We prove that maximal possible sizes of bipartite 1-planar graphs whose one partite set is much smaller than the other one tends towards $2n$ rather than $3n$. In particular, we prove that if the size of the smaller partite set is sublinear in n, then $|E| = (2 + o(1))n$, while the same is not true otherwise.

Acknowledgements. This work was supported by the Slovak Research and Development Agency under the contract No. APVV-0482-11, by the grants VEGA 1/0529/15 and KEGA 040TUKE4/2014.

Weak P-ideals

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A family $\mathcal{K}$ of subsets of an infinite set M is called an ideal on M if $\mathcal{K}$ is closed under subsets and finite unions, contains all finite subsets of the set M and $M \notin \mathcal{K}$. M. Katětov [2], M. Laczkovich and I. Recław [3] and R. Filipów and P. Szuca [1] found certain family of ideals, called weak P-ideals in [3], playing an important role in investigations of the ideal versions of convergence of real-valued functions. Similarly, weak P-ideal became crucial notion in our investigation of ideal version of quasi-normal convergence. In fact, weak P-ideals are exactly those ideals such that there is a sequence of functions converging pointwisely but not converging quasi-normally with respect to ideal. One way to describe the family of all weak P-ideals is to consider the ideal Fin × Fin on $\mathbb{N} \times \mathbb{N}$ defined by

$$\mathrm{Fin} \times \mathrm{Fin} = \{A \subseteq \mathbb{N} \times \mathbb{N};\ |\{n;\ |\{m;\ (n,m) \in A\}| = \aleph_0\}| < \aleph_0\}.$$

Then an ideal $\mathcal{J} \subseteq \mathcal{P}(\omega)$ is a weak P-ideal if and only if $\mathcal{J}$ does not contain an isomorphic copy of the ideal Fin × Fin.

Our talk is devoted to ideals on natural numbers $\mathbb{N}$ with particular emphasis on weak P-ideals and their two ideal generalization. We discuss the role they play in ideal version of quasi-normal convergence as well as their basic properties and characterizations.

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References

[1] Filipów, R. – Szuca, P.: *Three kinds of convergence and the associated $\mathcal{I}$-Baire classes*, J. Math. Anal. Appl. **391** (2012), 1–9.

[2] Katětov M.: *On descriptive classification of functions*, in: General Topology and its Relations to Modern Analysis and Algebra, Proceedings of the Third Prague Topological Symposium, Prague, 1972, 235–242.

[3] Laczkovich, M. – Recław, I.: *Ideal limits of sequences of continuous functions*, Fund. Math. **203** (2009), 39–46.

Program 16. Konferencie košických matematikov

Programme of the 16th Conference of Košice Mathematicians

Streda – Wednesday 25. 3. 2015

12^{30} – **Obed – Lunch**

14^{00} – **Otvorenie konferencie – Conference opening**

14^{05} – Erika Škrabuľáková (ÚRaIVP FBERG TU) *On the Problem in the Class of 1-Planar Bipartite Graphs*

14^{25} – Pavol Široczki (ÚMV PF UPJŠ) *Light Graphs in Planar Graphs of Large Girth*

14^{45} – Mária Maceková (ÚMV PF UPJŠ) *Structure of 3-Paths in Graphs with Bounded Maximum Average Degree*

15^{05} – Miroslava Šuličová (ÚMV PF UPJŠ) *Centralizers of Monounary Algebras and Green's Relations*

15^{25} – Anna Mišková (ÚMV PF UPJŠ) *Spectral Functions Density of Toeplitz Localization Operators*

15^{45} – **Občerstvenie – Coffee-break**

16^{15} – Lenka Halčinová (ÚMV PF UPJŠ) *Integration with Respect to Probabilistic-Valued Measures*

16^{35} – Veronika Kopčová (ÚMV PF UPJŠ) *MLE and QLS Method for Connection between Uniform and Serial Correlation Structure*

16^{55} – Jana Pócsová (ÚRaIVP FBERG TU) *Blended Learning In Mathematics Course*

17^{15} – Eva Oceľáková (ÚMV PF UPJŠ) *Modeling Practical Placement of Trainee Teachers to Schools*

17^{35} – Pavel Molnár (ÚMV PF UPJŠ) *The Geometric Interpretation of Solutions of the Inequalities and Their Systems with the Help of GeoGebra*

18^{00} – **Večera – Dinner**

Štvrtok – Thursday 26. 3. 2015

12^{00} – **Obed – Lunch**

14^{00} – Irena Jadlovská (KMTI FEI TU) *Oscillation Criteria for Fourth Order Trinomial Delay Differential Equations*

14^{20} – Jacek Chudziak (DM FNS U Rzeszów) *Scale Invariance of the Equivalent Utility Principle under Cumulative Prospect Theory*

14^{40} – Péter Körtesi (IoM U Miskolc) *Separation Properties in Topological Inverse Semigroups*

15^{00} – Tomáš Gregor (MÚ SAV KE) *Fourier Series as Multi-Stimuli and Operation with Them*

15^{20} – Vladimír Janiš (KM FPV UMB) *Fragmentation in Mathematics Education*

15^{40} – **Občerstvenie – Coffee-break**

16^{10} – Michaela Vrbjarová (ÚMV PF UPJŠ) *Maximum Colouring*

16^{30} – Sergej Ševec *New Criteria for Inscribability and Non-Inscribability of Polyhedra*

16^{50} – Michal Dečo (ÚMV PF UPJŠ) *Ideals on the Real Line*

17^{10} – Jaroslav Šupina (ÚMV PF UPJŠ) *Weak P-Ideals*

18^{00} – **Večera – Dinner**

Piatok – Friday 27. 3. 2015

8^{30} – Zuzana Chladná (FMFaI UK) *Challenges in modeling vaccination behaviour*

9^{20} – Rafał Kalinowski (AGH Kraków) *On Some Partitions and Decomposition of Graphs*

10^{10} – **Občerstvenie – Coffee-break**

10^{40} – Viktor Pirč (KMTI FEI TU Košice) *Matematika pre nevidiacich*

11^{30} – Vladimír Gazda (KF EkF TU Košice) *On an Equilibrium Price in a Complex Local Structure of the Buyers and Sellers*

12^{30} – **Obed – Lunch**

14^{00} – Peter Frolkovič (SvF STU Bratislava) *Rovnica advekcie vo výuke, výskume a aplikáciach*

14^{50} – Piotr Faliszewski (AGH Kraków) *How to Elect a Committee of Representatives and Live to See the Result*

15^{40} – **Občerstvenie – Coffee-break**

16^{10} – Katarína Žilková (KU Ružomberok) *Conceptions and Misconceptions about Geometric Shapes*

17^{00} – Roman Plch (MU Brno) *Nástroje pro tvorbu interaktivních výukových materiálů*

18^{30} – **Večera a spoločenský večer – Dinner & Party**

Sobota – Saturday 28. 3. 2015

8^{30} – Peter Szabó (LF TU) *High Performance Computing and Primes*

8^{50} – Ireneusz Krech (PedU Kraków) *On Some Nontransitive Relation*

9^{10} – Jozef Kiseľák (ÚMV PF UPJŠ) *On Integral Equations and Related Problems*

10^{00} – **Záver konferencie – Conference closing**

10^{10} – **Občerstvenie – Coffee-break**

11^{00} – **Obed – Lunch**

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