Jednota slovenských matematikov a fyzikov
Pobočka Košice

Prírodovedecká fakulta UPJŠ
Ústav matematických vied

Fakulta elektrotechniky a informatiky TU
Katedra matematiky a teoretickej informatiky

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18. Konferencia košických matematikov

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Predhovor

Vážení priatelia, milí hostia, kolegyne a kolegovia,

vitajte na 18. Konferencii košických matematikov. Túto konferenciu organizuje Jednota slovenských matematikov a fyzikov, pobočka Košice, v spolupráci s Ústavom matematických vied Prírodovedeckej fakulty UPJŠ, katedrami matematiky Technickej univerzity a pobočkou Slovenskej spoločnosti aplikovanej kybernetiky a informatiky pri KRVP BF TU v Košiciach. Konferencia sa koná, tak ako aj jej predchádzajúce ročníky, v útulnom prostredí Učebno-výcvikového zariadenia TU Košice – v Herľanoch.

Nápad organizovať konferenciu tohto typu vznikol v našej pobočke JSMF pod vedením prof. Jendroľa pred viac ako sedemnástimi rokmi. Bola za tým myšlienka, že ľudia profesionálne sa zaoberajúci matematikou v jej rôznych podobách (učitelia, vedci, aplikovaní matematici) a žijúci na východe Slovenska by mali mať možnosť sa pravidelnejšie stretávať, podeliť sa s rovnako „postihnutými“ kolegami o svoje radosti i starosti súvisiace s prácou matematika či matematikára; následne spoločne alebo s ďalšími spriaznenými dušami hľadať riešenia či východiská z problémov. Prípadne si vzájomne pomáhať a povzbudiť sa navzájom. Ďalej to bola predstava, že by malo ísť o serióznu konferenciu s kvalitným obsahom, najmä pozvanými prednáškami. Od začiatku boli na ňu pozývaní prednášajúci s cieľom, aby to boli či už zrelé alebo práve vychádzajúce kvalitné osobnosti, známe vo svojom prostredí, s cieľom dozvedieť sa nové veci, nadviazať nové či upevniť staré kontakty. Viaceré z týchto prednášok mali taký pozitívny ohlas, že ich autori boli pozvaní prednášať aj na iných konferenciách.

To, že Konferencia košických matematikov sa koná po 18. krát je len potvrdením, že tieto myšlienky našli úrodnú pôdu. Každoročne sme na nej mali skvelých prednášajúcich. Na výbere a príprave konferencie sa pracuje celý rok. O výbere pozvaných prednášajúcich sa v podstate rozhoduje na tradičnom každoročnom stretnutí výboru košickej pobočky JSMF s košickými profesormi matematiky a vedúcimi košických matematických pracovísk, vrátane riaditeľa Gymnázia na Poštovej ulici, ktoré má matematické triedy.

Po 17 rokoch sa čiastočne mení štruktúra konferencie. Prvý deň (štvrtok) je venovaný najmä mladým začínajúcim matematikom. Mnohí dnes už veľmi úspešní kolegovia mali svoje prvé verejné odborné či vedecké vystúpenie práve na našej konferencii. Vystúpenia mladých kolegov majú z roka na rok vyššiu úroveň, čo organizátorov veľmi teší. V piatok a v sobotu dopoludnia sa konajú najmä pozvané prednášky, aby sa na nich mohlo zúčastniť čo najviac účastníkov. Spoločenský piatkový večer je organizovaný tak, aby bolo možné v menších skupinách pri poháriku vínka predebatovať rôzne otázky.

Aj tento rok sa nám podarilo získať viacero výrazných osobností. Pozvanie prednášať prijali: doc. RNDr. Martin Mačaj, PhD. (KAGaDM, FMFI, UK Bratislava), doc. Mgr. Ján Mačutek, PhD. (KAMaS FMFI UK Bratislava), doc. RNDr. Mária Markošová, PhD. (OUI FMFI UK Bratislava), doc. RNDr. Miron Pavluš, CSc. (KMMaMI FM PU v Prešove), Dr. Suhadi Wido Saputro (Bandung Institute of Technology, Indonesia), RNDr. Ingrid Semanišinová, PhD. (UMV PF UPJŠ Košice), doc. RNDr. Roman Soták, PhD. (UMV PF UPJŠ Košice), doc. RNDr. Jan Šustek, Ph.D. (KM PF OU v Ostrave) a Prof. RNDr. Pavol Zlatoš, CSc. (KAGaDM, FMFI, UK Bratislava).

Prajeme vám príjemný pobyt v Herľanoch

Organizačný výbor: Ján Buša
Jozef Doboš
Róbert Hajduk

Obsah – Contents

Invited lectures

Improvements to Moore Bound for Vertex-Transitive Graphs of Given Degree and Diameter

Martin Mačaj

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The *Degree/Diameter Problem* is the problem of finding the largest order $n(\Delta, D)$ of a graph of maximum degree Δ and diameter D. The well-known Moore bound, $M(\Delta, D) = 1 + \Delta((\Delta - 1)^D - 1)/(\Delta - 2)$, provides a natural upper bound on $n(\Delta, D)$, and graphs that attain this bound are called Moore graphs. To avoid trivialities we will assume $\Delta \geq 3$ and $D \geq 2$, in which case Moore graphs are very rare. Any graph G of maximum degree Δ and diameter D (a (Δ, D)-graph) is said to have the *defect* $\delta(G) = M(\Delta, D) - |V(G)|$.

We present bounds on the number of cycles of length $2D + 1$ in graph with prescribed degree Δ, diameter D and defect δ which besides the term $\Delta(\Delta - 1)^D/2$ depend only on degree Δ and defect δ.

Using two classical number theory results due to Niven and Erdős, we prove that for any fixed degree $\Delta \geq 3$ and any positive integer δ, the order of a largest vertex-transitive Δ-regular graph of diameter D differs from the Moore bound by more than δ for (asymptotically) almost all diameters $D \geq 2$. We also obtain an estimate for the growth of this difference, or defect, as a function of D.

This talk is based on a joint work with G. Exoo, R. Jajcay and J. Širáň.

Acknowledgement. The author acknowledges the support by grants VEGA 1/0474/15 and APVV-15-0220.

Analysis of Partial-Sums Discrete Probability Distributions

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Consider partial-sums distribution P_x given by

$$P_x = \sum_{j=x}^{\infty} g(j) P_j^*, \qquad x = 0, 1, 2, \ldots, \tag{1}$$

with $\{P_j^*\}_{j=0}^{\infty}$ being the parent distribution and $\{P_x\}_{x=0}^{\infty}$ the descendant distribution (see [1, 2, 4]), both defined on nonnegative integers. Relations between some characteristics of parent and descendant distributions (i.e., between the one which is summed and the one which is the result of the summation), such as moments and probability generating functions, can be found in [2]. New results on this process of creating probability distributions will be presented.

First, we will focus on iterated partial summations.

$$\begin{aligned}
P_x^{(1)} &= c_1 \sum_{j=x}^{\infty} g(j) P_j^*, \qquad x = 0, 1, 2, \ldots \\
P_x^{(2)} &= c_2 \sum_{j=x}^{\infty} g(j) P_j^{(1)}, \qquad x = 0, 1, 2, \ldots \\
&\vdots \\
P_x^{(n)} &= c_n \sum_{j=x}^{\infty} g(j) P_j^{(n-1)}, \qquad x = 0, 1, 2, \ldots \\
&\vdots
\end{aligned}$$

In [3] it was shown that if $g(j) = p, \quad p \in (0, 1)$ (i.e., if it is a constant function), the geometric distribution is the limit of repeated partial summation for many parent distributions. Limit distributions were derived for some other choices of $g(j)$.

Second, we will consider parametrized partial summations. For the sake of simplicity we limit ourselves to discrete distributions with one parameter only. In [2] a necessary and sufficient condition of invariance under summation (1) is provided,

$$g(x) = 1 - \frac{P_x^*(a)}{P_{x+1}^*(a)}. \tag{2}$$

Under this condition, the parent distribution remains unchanged under (1), i.e., $P_x^* = P_x$, $x = 0, 1, 2, \ldots$ In order to emphasize the role of the parameter, we can use the notation $g(x) = g(x, a)$.

Now we define a modification of summation (1),

$$P_x = c \sum_{j=x}^{\infty} g(j, \lambda) P_j^*(a), \qquad x = 0, 1, 2, \ldots, \tag{3}$$

where the formula (2) is kept but parameter a was replaced with λ; c is an appropriate constant which ensures that $\{P_x\}_{x=0}^{\infty}$ is a proper distribution (i.e., it sums to 1).

The descendant distribution $\{P_x\}_{x=0}^{\infty}$ either depends on two parameters, λ and a, for $a \neq \lambda$, or the second parameter λ is "cancelled" by the normalization constant c. With respect to this property, it is possible to categorize every discrete distribution with one parameter into one of the two abovementioned classes. For example, the Poisson distribution generates under summation (3) a new distribution with two parameters, therefore it belongs to the first class. On the other hand, e.g., the geometric distribution belongs to the second class, i.e., it remains unaltered under summation (3).

Acknowledgement

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Complex Networks and Real World Phenomena

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Many real networks have nontrivial structure, which cannot be captured by classical Erdős-Renyi random graph models [1]. Unlike random graphs, real networks, such as Facebook, Internet, functional brain networks, transportation networks, networks of professional contacts are scale free, small world or even hierarchical structures. Even more, they are usually systems having many nodes and edges and are changing in time. Such real networks as Facebook, for example, grows in time. This is often the case of real systems. They are growing for sufficiently large time intervals, because the number of nodes coming to the system is far more greater then the number of nodes deleted from the system.

Real networks are difficult to visualize, and therefore their structure is described by the set of statistical measures [2]. Some of them are averages, such as average degree $\bar{k}$, average clustering coefficient $\bar{C}$ or average shortest path $\bar{l}$. More sophisticated measures are distributions, for example degree distribution $P(k)$ which measures normalized number of nodes having the degree k. Many real networks are scale free networks, characterized by the power law degree distribution

$$P(k) \propto k^{-\gamma} \qquad (4)$$

which is linear in the log-log plot. Scaling exponent γ is estimated as a slope of this line. Some of them are even scale free nets with hierarchical node organization [3]. Together with (4), they are also characterized by the power law dependence of the average clustering coefficient of the nodes with the degree k on k

$$C(k) \propto k^{-\delta}. \qquad (5)$$

Hierarchical scale free networks are therefore described by the two scaling exponents γ and δ.

The best possibility, of course, is to have a mathematical model of certain real network. Such network models are integro-differential dynamical equations capturing network changes in time. If the model is analytically solvable, the above mentioned averages and distributions are directly calculated from the model.

Here I present basic statistical measures and basic network models, together with the real world phenomena they describe. The simple Barabási - Albert model is explained in details, because it is easily solvable and other models are well understood on its basis [4]. To explain the structure of the real positional word web, our refinement of the Dorogovtsev - Mendes model is presented [5]. I also mention small world property in real scale free networks and present several models of growing networks producing scale free hierarchical nets [6]. In all of them the basic mechanism is the network growth not by the simple node addition, but by the pattern addition, where the pattern is either deterministic, or disturbed by some amount of uncertainty.

Several real situations modeled with a help of network models are also discussed. The knowledge about the structure of social networks helps one to suppress such events as heresy (in the past)[7], or disease spreading (today)[8]. The knowledge about the structure and dynamics of some biological networks is also useful. For example understanding functional brain networks, can help in Alzheimer disease diagnostics. Our studies on functional brain networks are described and the possibility to create a mathematical model of functional brain networks is discussed [9].

To conclude, I present necessary mathematical tools for the complex network analysis. I also discuss several real complex networks and show the possibility to capture their dynamics in a mathematical model.

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2D Macroscopic Simulation of Water Vapor and Porous Material Interaction

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The contribution deals with water vapor interaction in porous materials and with water vapor dynamics in a pore. A 2D macroscopic model is constructed and is solved by means of the variables separation method. The obtained solution will be compared with solution of 2D microscopic model.

This problem and methodology were published in the work [1]. Other aspects of the moisture drying problem were studied in [2], and [3].

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On Fractional Metric Dimension of Comb Product Graphs

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Let G be a simple connected graph. A vertex z in G *resolves* two vertices u and v in G if the distance from u to z is not equal to the distance from v to z. A set of vertices $R_G\{u,v\}$ is a set of all resolving vertices of u and v in G. For every two distinct vertices u and v in G, a *resolving function* f of G is a real function $f : V(G) \to [0,1]$ such that $f(R_G\{u,v\}) = \sum_{z \in R_G\{u,v\}} f(z)$ is at least 1. The minimum value of $|f| = \sum_{z \in V(G)} f(z)$ from all resolving functions f of G is called the *fractional metric dimension* of G. In this paper, we consider a graph which is obtained by the comb product between two connected graphs. In chemistry, some classes of chemical graphs can be considered as the comb product graphs. Let o be a vertex of H. The *comb product* between G and H, denoted by $G \rhd_o H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and identifying the i-th copy of H at the vertex o to the i-th vertex of G. For any connected graph G, we determine the fractional metric dimension of $G \rhd_o H$ where H is a connected graph of order m having a vertex of degree 1, 2, or $m-1$.

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Preparing Pre-Service Teachers for Classroom Practice

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We will discuss main areas of knowledge that are essential for a preparation of effective mathematics teacher. We have observed significant gaps in the pedagogical content knowledge (PCK) of our secondary pre-service teachers during the last years. This stimulated us to improve our course of Didactics of Mathematics. We will present our new approach which was mainly focused on the assessment of the lesson plans presented by the pre-service teachers. Our experience indicates that the authentic assessment focused on pre-service teachers' lesson plans and objectivised by five rubrics, tied with the content knowledge, and three types of PCK can help pre-service teachers to develop their PCK. The presented examples demonstrate a development of identification of didactic problems and misconceptions, precise selection of tasks and formulation of learning objectives.

Generalized Nonrepetitive Sequences on Arithmetic Progressions

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A sequence $S = s_1 \dots s_n$ is said to be *nonrepetitive* if no two adjacent blocks of S are identical. In 1906 Thue (see [4]) proved that there exist arbitrarily long nonrepetitive sequences over 3-element set of symbols. We study a generalization of nonrepetitive sequences involving arithmetic progressions introduced by Currie and Simpson (see [1]). We prove that for every $k \geq 0$, there exist arbitrarily long sequences over at most $k + 100\sqrt{k}$ symbols whose subsequences, indexed by arithmetic progressions with common differences from the set $\{1, \dots, k\}$, are nonrepetitive. This improves a previous bound of $2k$ obtained by Kranjc, Lužar, Mockovčiaková and Soták (see [3]). Our approach is based on a technique introduced recently by Grytczuk, Kozik and Micek (see [2]), which was originally inspired by a constructive proof of the Lovász Local Lemma due to Moser and Tardos.

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Real Numbers Expression Methods

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There are many ways how to express real numbers. The best known is the decadic expansion

$$x = \sum_{n=1}^{\infty} \frac{c_n}{10^n} \,. \tag{6}$$

Here, given a real number x, we are looking for the coefficients (digits) c_n. There is a restriction $c_n \in \{0, 1, \dots, 9\}$. With this restriction, every number $x \in [0, 1]$ can be expressed in the form (6). One can ask, for what numbers x the expression (6) is unique, how can one determine irrationality of x depending on the sequence c_n, and so on. These results are well known.

One can use another restriction on the sequence c_n. Then the question of existence of expansion (6) arises. Or the problem of uniqueness can be more complicated.

Instead of decadic expansion (6), one can use a general b-adic expansion for a given b with $2 \le b \in \mathbb{N}$, or even without this restriction on b. There are also other expansions, such as Cantor expansions, Engel expansions or continued fractions,

$$x = \sum_{n=1}^{\infty} \frac{c_n}{\prod\limits_{k=1}^{n} b_k} \,, \qquad x = \sum_{n=1}^{\infty} \frac{1}{\prod\limits_{k=1}^{n} c_k} \,, \qquad x = \frac{1}{c_1 + \frac{1}{c_2 + \cdots}} \,.$$

In all these expansion, given a real number x, we are looking for a sequence of coefficients c_n. In the talk we will discuss the problems of existence and uniqueness of the sequence c_n. We will also consider the sets of all representable x, depending on restrictions on c_n. Applications of particular types of expansion will also be presented.

The Problem of Consistency of the Foundations of Mathematics at the Beginning of the 21^{st} Century

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In the last decades of the 20^{th} and in the beginning of the 21^{st} century, in connection with still more frequent occurrences of algorithmically undecidable problems, increasing role of computer assisted proofs, and, in general, realizing the limits of mathematical knowledge caused by the phenomenon of the *complexity* as a barrier, which — though it can be shifted still farther away — can never be overcome completely, there have re-emerged, though in a slightly modified form, some questions dealing with the philosophy and the foundations of mathematics, which played such a crucial role in the development of the mathematics itself as well as of the opinions concerning the character of the mathematical and, more generally, the human knowledge as the whole, from the end of the 19^{th} till the half of the 20^{th} century.

In my lecture I will attempt at their comparison.

Conference contributions

Modern Trends in Teaching Mathematics

Beatrix Bačová

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Mathematics is an integral part not only of the scientific and technical world, but also of art, sociology, archeology, biology and sports. Its applications point to the fact that without the use of mathematics we would often fail to solve even simple life situations. Mathematics accompanies us in everyday situations – during doing shopping, travelling, working, but also playing games (e.g., billiards), and doing various hobbies (e.g., playing musical instruments). Mathematics teaches students how to think logically, how to look for the combinations when solving problems. It also teaches us to be accurate. Despite this fact, many people consider mathematics to be their lifelong problem. Primary, secondary as well as university teachers of mathematics frequently have to answer the questions: "Why do we need to learn mathematics?" "What use is mathematics in everyday life?" These question are especially surprising when asked by the university students. They should be clear about the importance of mathematics when they choose to continue in their studies of mathematics at the university. Therefore, to motivate students to study mathematics has become a difficult task for many teachers. They should know how to bring fun and enlightenment to the math class. There exist more ways how to make mathematical formulas, equations and principles interesting and challenging for our students. ICT (information and communication technologies) and various applications that students already have on their smartphones can be of great help to us. Also CLIL (Content and Language Integrated Learning) and Mathematical Corpus application in the teaching/learning process will strengthen the ability of our students to think logically, see phenomena in context and will support inter-subject relations.

Mathematical Signs and Symbols

Jozef Doboš

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ISO 80000-2:2009 is a standard describing mathematical signs and symbols developed by the International Organization for Standardization. The aim of my article is to introduce the Slovak translation (STN EN ISO 80000-2:2017).

A Benefit of Graph Theory in Economics and Technical Disciplines

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Graph theory has widespread use. It found its applications not only in mathematics, informatics and other natural sciences, but also in technical disciplines, earth and social sciences.

Here we want to point out its utilization in economics [1], [2] and technical disciplines and how our students are able to use it in order to solve different tasks they come in contact with now or later in their practice: logistics, risk management, credit scoring, churn and fraud detection ...

The emphasis is put on the interpretation of decision trees and explaination of their use in the connection with data mining in financial services. Via presented model situations we show that basics of graph theory have their accepted place in the engineering education.

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What are the Common Mistakes in Solving of the Probability Tasks of High School Students?

Tadeáš Gavala

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In teaching of the probability, teachers focus on the choice and solving problems related to everyday life. In this paper, we analyze the solutions of tasks with similar focus. These tasks were given to students as answer sheets. The aim was to explore the approaches of high school students to the solutions of probability tasks and identify the most common mistakes. In the introduction of paper, we highlight the important position of probability theory in the State educational program. The main part describes the aims, methods and organization of the research. We focus on the selected topics of probability and analyze solutions of several tasks. In conclusion, we summarize the partial findings. We also reflect how to eliminate shortcomings in the knowledge of students in teaching of the probability.

About the Use of Self-Asessment Rubrics

Timea Gábová

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Nowadays, more and more emphasis is putting on change in the education. An important instrument of this change could be a formative assessment. One aspect of the formative assessment, coming directly from the students, is student self-evaluation. The paper is focused on the self-assessment rubric as a tool of the formative assessment and it is focused on the inclusion of the self-assessment rubric in the educational process, specifically in the one topic teaching of high school students. The paper also analyzes the impact of the self-assessment rubrics on the learning process.

The Story of Concatenation

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We study the state complexity of the concatenation operation on regular languages represented by deterministic and alternating finite automata. For deterministic automata, we show that the upper bound $m2^n - k2^{n-1}$ on the state complexity of concatenation can be met by ternary languages, the first of which is accepted by an m-state DFA with k final states, and the second one by an n-state DFA with ℓ final states for arbitrary integers m, n, k, ℓ with $1 \leq k \leq m-1$ and $1 \leq \ell \leq n-1$. In the case of $k \leq m-2$, we provide appropriate binary witnesses. In the case of $k = m-1$ and $\ell \geq 2$, we provide lower bound which is smaller than the upper bound just by one. We conjecture that this lower bound is tight in the binary case. We use our binary witnesses for concatenation on deterministic automata to describe binary languages meeting the upper bound $2^m + n + 1$ for the

concatenation on alternating finite automata. This solves an open problem stated by Fellah, Jürgensen, and Yu [1990, Constructions for alternating finite automata, Intern. J. Computer Math. **35**, 117–132]. At the end we show an upper bound $m+n+1$ and a lower bound $m+n-1$ for concatenation on unary alternating finite automata.

Acknowledgment. The present work was supported by the VEGA grant 2/0084/15 and grant APVV-15-0091.

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Complement on Free and Ideal Languages

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We study nondeterministic state complexity of the complement operation on the classes of prefix-free, suffix-free, factor-free and subword-free languages and on the class of ideal languages. For the cases prefix-free and suffix-free we improve the lower bound, and improve the upper bound for suffix-free languages in the binary case. In all other cases, we found tight bounds for sufficient alphabet sizes.

Using LaTeX for Creating Mathematical Animations

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One of the possibilities how to explain solving of mathematical examples is to use animations. There are many tools for creating animations. In this contribution we present creating animations with the LaTeX animate package. The animate package creates JavaScript driven PDF animations from the set of image files, inline graphics or typeset text [2]. We create animations using inline PGF/TikZ generated pictures [3]. PGF (Portable Graphics Format) and TikZ (TikZ ist kein Zeichenprogramm) are packages for creating graphics integrated with LaTeX and Beamer [1]. We will present LaTeX animations created for the basic undergraduate course Mathematics 1 at the Faculty of Mining, Ecology, Process Control and Geotechnology, Technical University of Košice.

Acknowledgement. The present work was supported by Slovak Research and Development Agency under the contracts No. APVV-14-0892, VEGA grants No. 1/0529/15 and No. 1/0908/15.

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Constructivist Approaches to the Teaching of Statistics and Probability

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Maths is not usually very favorite subject in schools of all levels and types. Statistics seems to be moreover one of the least popular branches of mathematics in spite of being very useful in sciences, technics, economy etc.

There are many approaches that can improve the quality and efficiency of educational process.

One of them is called constructivist approach and it emphasizes active role of students, importance of their inner properties, their interactivity with environment and society. Students are led to invented new knowledge themselves step by step by simple graduated tasks, examples and problems.

Another interesting attitude to maths teaching is called method of genetic parallels. It uses similarity between ontogenetic and phylogenetic development. New knowledge are presented and discussed in the same order as they were discovered in history.

Some e-learning tools can be successfully used in the education process. It can be supported by some special programmes as well as by some sophisticated websites, applets or hypertext.

It was several times proved that the motivation presents the basic moment in education. The positive attitude plays the key role in teaching/learning of any discipline.

We would like to present how some of these attitudes elements can be used in teaching of statistics and probability calculus.

2D Microscopic Simulation of Water Vapor and Porous Material Interaction

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In various fields of science, technology, environmental protection, construction, actual issues are the study of the interaction processes porous materials with various substances. Especially relevant from the point of view of ecology and protection environmental studies are the processes of interaction of porous materials with water in the liquid and gaseous phase.

We mainly use macroscopic approaches to describe the properties, for example, of water vapour in the pore. In this contribution we consider a microscopic model of the behaviour of water vapour inside an isolated pore, constructed in the framework of the molecular dynamics approach [1]. The model is based on the Newton's classical mechanics. That means, the motion of each water molecule interacting with both other water molecules and with walls of the pore is described by differential equations.

In the contribution we consider 2D model of pore with dimensions 1 $\mu m \times 1$ μm. We do calculations for the case, when outer temperature is 25°C. A density and number of particles depends on temperature. We are dedicated two processes – drying process and wetting process. The code is written in CUDA C. It is implemented on heterogeneous computing cluster HybriLIT.

We explore the evolution of the water vapour system in time. Depending on external conditions with respect to pores, the system evolves to different equilibrium states, which are characterized by different values of macroscopic characteristics such as temperature, density, and pressure.

We compared the results of molecular dynamics simulation with the results calculations on the basis of a macroscopic diffusion model. The both models give a consistent description of water vapour and a pore interaction in 2D case.

References

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On Some Aspects in the Mathematical Education at the Technical University

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In this paper, we discuss some didactic principles related to the mathematical education at the Faculty of Mining, Ecology, Process Control and Geotechnology, Technical University of Košice. We show several ways how to students become familiar with some basic notions from the mathematical analysis, particularly notions concerning extrema of functions of two variables. We focus on the visualization of these notions and their subsequent categorization into the knowledge system of students.

Acknowledgement. The present work was supported by Slovak Research and Development Agency under the contracts No. APVV-14-0892, VEGA grants No. 1/0529/15 and No. 1/0908/15.

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[1] Takáč, M.: *Lokálne extrémy funkcie* [Online]. Retrieved April 3, 2017 from `http://michaltakac.com/math-project/local-extrema-multiple-variables/`

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Ideal Convergences and Sequence Selection Principle

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Our motivation to study wQN-space and its modifications was the paper [1] by Bukovský, Das and Šupina. They obtained relations between space of all continuous functions on X and some kinds of quasi-normal spaces. Especially, particular ideal version of quasi-normal space, $(\mathcal{I}, s\mathcal{J})$wQN-space, is helpful to describe sequence selection principle modified using the ideal convergence.

In the presentation we summarize several results by [1] about this topic and add our ones, mainly the equivalence between two types of $(\mathcal{I}, s\mathcal{J})$wQN-spaces described by different control sequences.

Moreover, there are several other authors as Filipów and Staniszewski [2], Šupina [3] etc, who study similar problems and we mention their results as well.

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[1] Bukovský L., Pratulananda D., Šupina J.: *Ideal quasi-normal convergence and related notions*, Colloq. Math. 3506 (2017), 265–281.

[2] Filipów R., Staniszewski M.: *Pointwise versus equal (quasi-normal) convergence via ideals*, J. Math. Anal. Appl. 422 (2015) 995–1006.

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Program 18. Konferencie košických matematikov

Programme of the 18th Conference of Košice Mathematicians

Štvrtok – Thursday 20. 4. 2017

12^{30} – **Obed – Lunch**

14^{00} – **Otvorenie konferencie – Conference opening**

14^{05} – Andrea Mojžišová (ÚRaIVP FBERG TU) *Using LaTeX for Creating Mathematical Animations*

14^{25} – Jana Pócsová (ÚRaIVP FBERG TU) *On Some Aspects in the Mathematical Education at the Technical University*

14^{45} – Erika Fecková Škrabuľáková (ÚRaIVP FBERG TU) *A Benefit of Graph Theory in Economics and Technical Disciplines*

15^{05} – Peter Mlynárčik (MÚ SAV) *Complement on Free and Ideal Languages*

15^{30} – **Občerstvenie – Coffee-break**

16^{00} – Michal Hospodár (MÚ SAV) *The Story of Concatenation*

16^{25} – Viera Šottová (ÚMV PF UPJŠ) *Ideal Convergences and Sequence Selection Principle*

16^{50} – Tadeáš Gavala (ÚMV PF UPJŠ) *What are the Common Mistakes in Solving of the Probability Tasks of High School Students?*

17^{15} – Timea Gábová (ÚMV PF UPJŠ) *About the Use of Self-Asessment Rubrics*

18^{00} – **Večera – Dinner**

Piatok – Friday 21. 4. 2017

9^{00} – Ján Mačutek (FMFI UK Bratislava) *Analysis of Partial-Sums Discrete Probability Distributions*

9^{50} – Martin Mačaj (FMFI UK Bratislava) *Improvements to Moore Bound for Vertex-Transitive Graphs of Given Degree and Diameter*

10^{40} – **Občerstvenie – Coffee-break**

11^{10} – Miron Pavluš (FM PU Prešov) *2D Macroscopic Simulation of Water and Porous Material Interaction*

11^{35} – Mária Popovičová (FM PU Prešov) *2D Microscopic Simulation of Water and Porous Material Interaction*

12^{00} – **Obed – Lunch**

13^{30} – Mária Markošová (KAI FMFI UK Bratislava) *Complex Networks and Real World Phenomena*

14^{20} – Pavol Zlatoš (FMFI UK Bratislava) *The Problem of Consistency of the Foundations of Mathematics at the Beginning of the 21*st *Century*

15^{10} – **Občerstvenie – Coffee-break**

15^{40} – Jan Šustek (PF OU Ostrava) *Methods of Expression of Real Numbers*

16^{30} – Roman Soták (ÚMV PF UPJŠ) *Generalized Nonrepetitive Sequences on Arithmetic Progressions*

17^{20} – Ingrid Semanišinová (ÚMV PF UPJŠ) *Preparing Pre-Service Teachers for Classroom Practice*

18^{30} – **Večera a spoločenský večer – Dinner & Party**

Sobota – Saturday 22. 4. 2017

9^{00} – Beatrix Bačová (SvF ŽU Žilina) *Modern Trends in Teaching Mathematics*

9^{20} – František Mošna (PF UK Praha) *Constructivist Approaches to the Teaching of Statistics and Probability*

9^{40} – Suhadi Wido Saputro (Bandung IoT, Indonesia) *On Fractional Metric Dimension of Comb Product Graphs*

10^{10} – Jozef Doboš (ÚMV PF UPJŠ) *Mathematical Signs and Symbols*

10^{30} – **Záver konferencie – Conference closing**

10^{35} – **Občerstvenie – Coffee-break**

11^{00} – **Obed – Lunch**

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