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On cyclic chromatic number of plane graphs

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Abstract. A cyclic colouring of a plane graph G embedded in a surface is a vertex colouring of G in which any two distinct vertices sharing a face receive distinct colours. The cyclic chromatic number $\chi_c(G)$ of G is the smallest number of colours in a cyclic colouring of G. It is conjectured that $\chi_c(G) \leq \lfloor \frac{3}{2}\Delta^*(G) \rfloor$ for any planar graph G with a maximum face size $\Delta^*(G)$. Sanders and Zhao in [3] proved that $\chi_c(G) \leq \lceil \frac{5}{3}\Delta^*(G) \rceil$ for any planar graph G. Their proof uses the Discharging Method and a knowledge of structural properties of a hypothetical minimal counterexample. In the present paper Lemma 2.5 of [3] about structural properties of a hypothetical minimal counterexample is generalized.

1 Introduction

A *cyclic colouring* of a plane graph G is a such colouring of vertices of G that whenever two distinct vertices share a face, they receive distinct colours. The cyclic chromatic number $\chi_c(G)$ of the graph G is the minimum number of colours in a cyclic colouring of G.

This invariant was introduced (in the dual form) by Ore and Plummer [2]. First upper bound $2\Delta^*(G)$ (where $\Delta^*(G)$ denotes a maximum face size of a graph G) was given by Ore and Plummer [2], later it was improved to $\lceil \frac{9}{5}\Delta^*(G) \rceil$ by Borodin, Sanders and Zhao [1] and the best known bound $\lceil \frac{5}{3}\Delta^*(G) \rceil$ is due to Sanders and Zhao [3].

On the other hand, there is an infinite family of plane graphs G satisfying $\chi_c(G) = \lfloor \frac{3}{2}\Delta^*(G) \rfloor$. It is conjectured that $\chi_c(G) \leq \lfloor \frac{3}{2}\Delta^*(G) \rfloor$ for any plane graph G.

Upper bounds for the cyclic chromatic number are so far all based on the well-known Discharging Method and it is hard to imagine that it will be otherwise for possible future improvements. Discharging Method is closely related

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to information on structural properties of a hypothetical minimal counterexample, namely to reducible configurations (those which cannot appear in a minimal counterexample). In this article some reducible configurations are established by considering cyclic colourings of a plane graph G with $\left\lceil (1+\frac{p}{q})\Delta^*(G) \right\rceil$ colours, where $\frac{1}{2} < \frac{p}{q} \leq \frac{2}{3}$.

2 Preliminaries

Let $G = (V, E, F)$ be a simple 2-connected plane graph. The *degree* $\deg(x)$ of $x \in V \cup F$ is the number of edges incident with x. A vertex of degree k is a *k-vertex*, a vertex of degree at least k is a *$(k+)$-vertex*, a face of degree k is a k-face. By $V(x)$ we denote the set of all vertices incident with $x \in E \cup F$; similarly, $F(y)$ is the set of all faces incident with $y \in V \cup E$. If $e \in E$, $F(e) = \{f_1, f_2\}$ and $\deg(f_1) \leq \deg(f_2)$, the pair $(\deg(f_1), \deg(f_2))$ is called the *type* of e. A (d_1, d_2)-*neighbour* of a vertex x is a vertex y such that the edge xy is of type (d_1, d_2). Let v be a vertex of degree n. Consider a sequence $(f_1, \ldots, f_n)$ of faces incident with v in a cyclic order around v (there are altogether $2n$ such sequences) and the sequence $D = (d_1, \ldots, d_n)$ in which $d_i = \deg(f_i)$ for $i \in [1, n]$. The sequence D is called the *type* of the vertex v provided that it is the lexicographical minimum of the set of all such sequences corresponding to v. A vertex x_1 is *cyclically adjacent* to a vertex $x_2 \neq x_1$ if there is a face f with $x_1, x_2 \in V(f)$. The *cyclic neighbourhood* $N_c(x)$ of a vertex x is the set of all vertices that are cyclically adjacent to x and the *closed cyclic neighbourhood* of x is $\bar{N}_c(x) := N_c(x) \cup \{x\}$. The *cyclic degree* of x is $\mathrm{cd}(x) := |N_c(x)|$. A cyclic colouring $\varphi : V \to C$ is called a k-colouring, if $|C| = k$ (where elements of C are called colours of the mapping φ).

Let p, q be positive integers, $\frac{1}{2} \leq \frac{p}{q}$. Suppose that there is a plane graph G of maximum face size $\Delta^*(G) = d$ which has no cyclic k-colouring, $k = \lceil (1+\frac{p}{q}) \cdot d \rceil$. Let such a graph with the smallest possible number of edges be called a (d, k)-*minimal graph.* A (d, k)-*reducible configuration* is a configuration that does not appear in any (d, k)-minimal graph.

The following reducibility lemma is a standard observation in the context of cyclic k-colourings (for appropriate k and any d), cf. [3]:

Lemma 1 1. *A (d, k)-minimal graph is 2-connected.*
2. *An edge of type (d_1, d_2) with $d_1 + d_2 \leq d + 2$ is (d, k)-reducible.*
3. *A vertex x with $\mathrm{cd}(x) < k$ is (d, k)-reducible.*

Sanders and Zhao in [3] defined a common vertex of two faces as follows: Given two faces f_1, f_2 of degree at least 4 of a plane graph G, let a *common vertex of f_1 and f_2* be a vertex incident with each of them which is either a vertex of degree 2, a vertex of degree 3 incident with a face of degree 3, or a vertex of degree 4 incident with two faces of degree 3. This concept of a common vertex is different from the conventional one. Note that if v is a common vertex of f_1 and f_2, then all of the vertices cyclically adjacent to v are incident with either f_1 or f_2. Also, let a *common vertex* be a vertex which is a common vertex of two faces of G. An *uncommon vertex* is one which is not common.

3 Results

The following lemma generalizes Lemma 2.5 of [3]:

Lemma 2 *Let p, q be positive integers such that $\frac{1}{2} < \frac{p}{q} \leq \frac{2}{3}$, let G be a plane graph with $\Delta^*(G) = d$. Let f_1, f_2, f_3 be three faces of the graph G, let $n_{i,j}$ be the number of common vertices of f_i and f_j. Also, let $n := 1$ if there is a vertex incident with all of f_1, f_2, f_3, else let $n := 0$. If $n_{1,2} \geq 1$ and $n_{1,3} + n_{2,3} + n \geq d - 1 + \frac{1-dp}{q}$, then this configuration is $(d, \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil)$-reducible.*

Proof. Suppose that G is a $(d, \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil)$-minimal graph with faces f_1, f_2, f_3 as in the statement. Let x be a common vertex of f_1 and f_2. Let H be G with an edge incident with x contracted. Since $\Delta^*(H) \leq d$, H has a cyclic $\left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$-colouring.

Without loss of generality, there is a common vertex y of f_1 and f_3 whose colour does not appear in f_2. (Note that by definition of a common vertex, it is clear that $x \neq y$.) This is true, for otherwise, for $i \in \{1, 2\}$, each of the $n_{i,3}$ colours on the common vertices of f_i and f_3 appears on the border of f_{3-i} and thus appear twice in $N_c(x)$. Note that $\mathrm{cd}(x) \leq 2d - 2$ and thus x sees at most $2d - 2 - (n_{1,3} + n_{2,3}) \leq 2d - 2 + 1 - d + \frac{dp-1}{q} + 1 = (1 + \frac{p}{q}) \cdot d - \frac{1}{q} < \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$ colours. Thus x may be coloured by a colour different from its cyclic neighbours, contradicting the minimality of G.

Suppose there is also a common vertex z of f_2 and f_3 whose colour does not appear in f_1. Now there are at least $\frac{pd}{q}$ colours which do not appear in f_3. Each of those colours appears in f_1, or else if c is a colour in neither f_1 nor f_3, then recolouring x with the colour on y and recolouring y with c gives a cyclic $\left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$-colouring of G. Symmetrically, arguing with z in place of y, each of those colours appears in f_2. Thus, at most $d(1 - \frac{p}{q})$ colours appear in f_2 which do not appear in f_1. It follows that x sees at most $d + d(1 - \frac{p}{q}) < \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$ colours. As before, x may be coloured to give a cyclic $\left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$-colouring of G.

The final case is to suppose that each of the $n_{2,3}$ colours appearing at the common vertices of f_2 and f_3 also appears in f_1. Clearly, none of these colours appears among the common vertices of f_1 and f_2. Thus, $\mathrm{cd}(y) \leq 2d - |V(f_1) \cap V(f_3)|$, and since x is uncoloured, including the colour on y itself, y sees at most $2d - |V(f_1) \cap V(f_3)| - n_{2,3} - 1 \leq 2d + 1 - d + \frac{dp-1}{q} - 1 = (1 + \frac{p}{q}) \cdot d - \frac{1}{q} < \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$ colours. Thus x may be coloured with the colour on y, and then y may be recoloured with a colour it does not see to give a cyclic $\left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$-colouring of G. ■

Lemma 3 *Let p, q be positive integers such that $\frac{1}{2} < \frac{p}{q}$, let G be a plane graph with $\Delta^*(G) = d$. Let $x_0 \in V(G)$ be a vertex of type $(3, d, d_1, d)$, where $d_1 < 2\left\lceil \frac{pd}{q} \right\rceil - d + 4$, or of type $(3, d, d)$. Let x_1, x_2 be neighbours of x_0 incident with a 3-face h. Let $f_i \neq h$ be the face incident with the edge x_0x_i, $i \in \{1, 2\}$ and let $f_3 \neq h$ be the face incident with the edge x_1x_2. Let $n_{i,j}$ be the number of common vertices of f_i and f_j. If $n_{1,3} \geq 1$ and there is a common vertex v of faces f_2 and f_3 with $\mathrm{cd}(v) \leq \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$, then this configuration is $(d, \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil)$-reducible.*

Proof. Suppose that G is a $(d, \left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil)$-minimal graph with vertices x_0, x_1, x_2, v and faces f_1, f_2, f_3 as in the statement. Let H be G with an edge incident with v contracted. Since $\Delta^*(H) \leq d$, H has a cyclic $\left\lceil (1 + \frac{p}{q}) \cdot d \right\rceil$-colouring $\varphi : V(H) \to C$. This colouring will be used to find a cyclic colouring $\psi : V(G) \to C$. If not stated explicitly otherwise, we put $\psi(u) = \varphi(u)$ for any $u \in V(G) - \{v\}$.

If there is a colour $c \in C - \varphi(N(v))$, then we put $\psi(v) = c$, else v sees every colour exactly once. If there is $i \in \{0, 1\}$ and a colour $c \in C - \varphi(\bar{N}(x_i))$, then we put $\psi(x_i) = c$ and $\psi(v) = \varphi(x_i)$, else both x_0, x_1 see every colour at least once. Then, because of the structure of this configuration, it holds:

1. If $\deg(x_0) = 4$, then there are at least $2\left\lceil \frac{pd}{q} \right\rceil - 1$ colours on at most $d + d_1 - 5$ vertices. Hence $2\left\lceil \frac{pd}{q} \right\rceil - 1 \leq d + d_1 - 5$ and $2\left\lceil \frac{pd}{q} \right\rceil + 4 - d \leq d_1$, a contradiction.

2. If $\deg(x_0) = 3$, then there are at least $2\left\lceil \frac{pd}{q} \right\rceil - 2$ colours on at most $d - 3$ vertices. Hence $2\left\lceil \frac{pd}{q} \right\rceil - 2 \leq d - 3$ and, since $\frac{1}{2} < \frac{p}{q}$, $d + 1 < 2\left\lceil \frac{pd}{q} \right\rceil + 1 \leq d$, a contradiction. ■

4 An application

Lemma 4 *The configuration C_i of Fig. i, $i \in \{1, 2\}$ (where encircled numbers represent degrees of corresponding vertices and vertices without degree specification are of an arbitrary degree) is $(8, 13)$-reducible.*

Proof. Let G be a $(8, 13)$-minimal graph. Let G contain a configuration C_i, $i \in \{1, 2\}$. Let H be G with an edge incident with v contracted. Since $\Delta^*(H) \leq 8$,

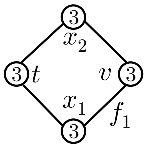

Fig. 1: deg(f_1)=7

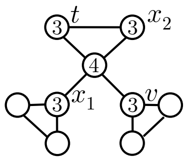

Fig. 2

H has a cyclic 13-colouring $\varphi : V(H) \to C$. This colouring will be used to find a cyclic colouring $\psi : V(G) \to C$. If not stated explicitly otherwise, we put $\psi(u) = \varphi(u)$ for any $u \in V(G) - \{v\}$.

If there is a colour $c \in C - \varphi(N(v))$, then we put $\psi(v) = c$, else v sees every colour exactly once. If there is a colour $c \in C - \varphi(\bar{N}(x_1))$, then we put $\psi(x_1) = c$ and $\psi(v) = \varphi(x_1)$, else also x_1 sees every colour exactly once.

Case $i = 1$: Then there is either a colour $c \in C - \varphi(\bar{N}(x_2))$ and we put $\psi(x_2) = c$ and $\psi(v) = \varphi(x_2)$ or a colour $c \in C - \varphi(\bar{N}(t))$ and we put $\psi(t) = c$ and $\psi(v) = \varphi(t)$.

Case $i = 2$: Then there is a colour $c \in C - \varphi(\bar{N}(x_2))$ and we put $\psi(x_2) = c$ and $\psi(v) = \varphi(x_2)$. ∎

Theorem 5 *Let H be a plane graph with $\Delta^*(H) = 8$. Then $\chi_c(H) \leq 13$.*

Proof. By contradiction. Let G be a $(8, 13)$-minimal graph. For any vertex $v \in V(G)$ let $c_0(v) = 1 - \frac{\deg(v)}{2} + \sum_{f \in F(v)} \frac{1}{\deg(f)}$ be the *initial charge* of vertex v. Then, using Euler's formula and the handshaking lemma, it is easy to see that $\sum_{v \in V} c_0(v) = 2$. If a vertex v is of type $(d_1, \ldots, d_n)$, then $c_0(v) = \gamma(d_1, \ldots, d_n) = 1 - \frac{n}{2} + \sum_{i=1}^{n} \frac{1}{d_i}$. Clearly, if π is a permutation of the set $\{1, \cdots, n\}$, then $\gamma(d_{\pi(1)}, \ldots, d_{\pi(n)}) = \gamma(d_1, \ldots, d_n)$.

A vertex $v \in V(G)$ is *positive* if $c_0(v) > 0$, otherwise it is nonpositive. For a vertex $v \in V$ let $n(v)$ denote the number of all positive neighbours of v and let $n_{4+}(v)$ denote the number of all neighbours of v of degree at least 4.

Note that because of Lemma 1.3 it holds that $\delta(G) \geq 3$.

Now let us state the following redistribution rules leading from c_0 to c_5 (where the coordinate i of a rule Ri means that Ri is used when passing from c_{i-1} to c_i):
R1 A vertex x of type $(4, 7, 8)$ with $n_{4+}(x) = 0$ sends an amount $c_0(x)$ to its $(4, 7)$-neighbour.
R2 A $(4+)-$vertex v sends an amount $-c_1(x)$ to its neighbour x of type $(4, 7, 8)$.
R3 A $(4+)-$vertex v sends an amount $-c_2(x) = -c_0(x)$ to its $(8, 8)$-neighbour x of type $(3, 8, 8)$.
R4 A $(5+)-$vertex v sends an amount $-c_3(x)$ to its $(3, 8)$-neighbour x of type $(3, 8, 8)$ with $c_3(x) > 0$.
R5 A 4-vertex v sends an amount $\frac{c_4(v)}{n(v)}$ to its $(3, 8)$-neighbour x of type $(3, 8, 8)$ with $c_4(x) > 0$.

Let $v \in V(G)$, denote $n = \deg(v)$ and let us show that $c_5(v) \leq 0$, which contradicts Euler's formula.

(1) If $n \geq 6$, then $c_5(v) \leq c_0(v) + n \cdot \gamma(3,8,8) \leq 1 - \frac{n}{2} + \frac{n}{6} + \frac{n}{16} + \frac{n}{12} = 1 - \frac{3n}{16} \leq 0$.

(2) If $n = 5$, then $c_5(v) \leq c_0(v) + 5 \cdot \frac{1}{12} \leq \gamma(3,8,3,8,8) + \frac{5}{12} = -\frac{1}{24} \leq 0$.

(3) If $n = 4$, then $c_0(v) \leq \gamma(3,8,3,8) = -\frac{1}{12}$ and $c_1(v) = c_0(v)$. Then if $n(v) = 0$, then $c_5(v) = c_0(v) \leq 0$, else some of rules R2, R3, R5 was applied on v. Clearly, if R5 was used, then $c_5(v) = 0$.

(31) If R2, but not R3 (and R5) was applied on v, then $c_5(v) = c_2(v) \leq c_0(v) + 4 \cdot 2\gamma(4,7,8) \leq \gamma(3,7,4,8) + 8 \cdot \frac{1}{56} = -\frac{1}{168} \leq 0$.

(32) If R3, but not R2 (and R5) was applied on v, then, $c_5(v) = c_3(v) \leq \max\{\gamma(3,8,8,8) + 2 \cdot \frac{1}{12}, \gamma(8,8,8,8) + 4 \cdot \frac{1}{12}\} = -\frac{1}{8} \leq 0$.

(33) If both R2 and R3 (but not R5) were used for v, then $c_5(v) = c_3(v) \leq c_0(v) + 3 \cdot \frac{1}{12} + 2 \cdot \frac{1}{56} \leq \gamma(4,7,8,8) + \frac{2}{7} = -\frac{1}{14} \leq 0$.

(4) If $n = 3$, then either $c_5(v) = c_0(v) \leq 0$ or $c_0(v) > 0$ and v is either of type $(4,7,8)$ or of type $(3,8,8)$.

(41) Let v be of type $(4,7,8)$. If $n_{4+}(v) > 0$, then, by R2, $c_5(v) = c_2(v) = 0$. Otherwise $c_5(v) = c_1(v) = 0$ by R1 and configuration C_1.

(42) Now let v be of type $(3,8,8)$. If its $(8,8)$-neighbour is of degree at least 4 or at least one of its $(3,8)$-neighbours is of degree at least 5, then, by R3 or R4, we have $c_5(v) = c_4(v) \leq 0$. Otherwise, by Lemma 3, at least one of $(3,8)$-neighbours of v is of degree 4. Denote this vertex as z and denote the $(3,8)$-neighbour of v distinct from z as y.

(421) If also $\deg(y) = 4$, then, by Lemma 3, at least one of y, z is not of type $(3,8,3,8)$. Therefore its charge after R4 is at most $\max\{\gamma(3,8,4,8) + 2 \cdot 2 \cdot \frac{1}{56}, \gamma(3,8,5,8), \gamma(3,8,6,8), \gamma(3,8,7,8) + 2 \cdot 2 \cdot \frac{1}{56}, \gamma(3,8,8,8) + 2 \cdot \frac{1}{12}\} = -\frac{2}{21}$ and so $c_5(v) \leq \frac{1}{12} - \frac{2}{21} \leq 0$.

(422) If $\deg(y) = 3$, then z is not of type $(3,8,5-,8)$ (by Lemma 3). Then, by rules, Lemma 3 and configuration C_2 $c_4(z) \leq \max\{\gamma(3,8,6,8), \gamma(3,8,7,8) + 2 \cdot 2 \cdot \frac{1}{56}, \gamma(3,8,8,8) + \frac{1}{12}\} = -\frac{5}{24}$ and so $c_5(v) \leq \frac{1}{12} - \frac{1}{2} \cdot \frac{5}{24} \leq 0$. ■

References

[1] O.V. Borodin, D. P. Sanders, Y. Zhao, Cyclic colorings and their generalizations, Discrete Math. **203** (1999) 23–40

[2] O. Ore and M.D. Plummer, Cyclic coloration of plane graphs, in: Recent Progress in Combinatorics (Proceedings of the Third Waterloo Conference on Combinatorics, Academic Press, New York 1969) 287–293

[3] D.P. Sanders and Y. Zhao, A new bound on the cyclic chromatic number, J. Combin. Theory Ser. B **83** (2001) 102–111

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