Discrete Mathematics

Vertex colourings of a graph induced by edge or total colourings
supervisor: prof. RNDr. Mirko Horňák, CSc.
study form: full time
Annotation: An edge colouring of a graph can in a natural way induce a vertex colouring of that graph (e.g. by means of the set or the multiset of colours of edges incident to a vertex, or else, if colours are positive integers, by means of the sum or the product of numbers present at a vertex). A total colouring assigns colours to edges and vertices as well, and so possibilities how to use it to induce a vertex colouring are even more rich. One can impose on the induced colouring different requirements (to be proper, to be surjective, …) and to look for the minimum possible number of colours in an original colouring that enables to fulfill the involved requirement.

Facially proper colorings of plane graphs
supervisor: prof. RNDr. Mirko Horňák, CSc.
consultant: RNDr. Igor Fabrici, Dr. rer. nat.
study form: full time
Annotation: By Vizing's theorem (1964), the edges of every graph may be properly colored using $\Delta+1$ colors, where Δ denotes the maximum degree of a graph. This bound is best possible and, moreover, Δ colors are necessary for a proper edge coloring of an arbitrary graph. In a proper face coloring of a plane graph it is allowed to color two faces with exactly one common vertex (but without common edge, i.e. two non-adjacent faces) by the same color. Thus, it is natural to consider an edge coloring (perhaps extended by a coloring of vertices and/or faces), in which only (facially adjacent) edges following in the clockwise order of edges around the common vertex (this order is given by an embedding of the involved graph in the plane) receive different colors. It is known that every plane graph has an edge coloring using 4 colors, a coloring of edges and vertices as well as a coloring of edges and faces using 6 colors (Fabrici, Jendroľ, Voigt, 2016), a coloring of edges, vertices, and faces using 8 colors (Fabrici, Jendroľ, Vrbjarová, 2016), in which facially adjacent edges (adjacent vertices, adjacent faces, and incident elements) are colored by different colors. The aim of the thesis is to study known types of edge colorings of plane graphs, where assigning different colors will be required only for facially adjacent edges.

Colorings of graphs embedded into compact surfaces
supervisor: prof. RNDr. Stanislav Jendroľ, DrSc.
study form: full time
Annotation: There are several well studied types of colorings of plane graphs where constrains on colorings are given by faces. In the proposed project analogous colorings will be investigated for graphs embedded into compact surfaces.

Clones of congruence compatible functions
supervisor: doc. RNDr. Miroslav Ploščica, CSc.
study form: full time
Annotation: A function on an algebraic structure is called (congruence) compatible if it preserves all congruences.Such functions are a generalization of polynomials and have similar properties. The set of all compatible functions is a clone (i.e. is closed under composition). The main topic of the research will be finding the generators of this clone, for algebraic structures of various types.

Generalized colorings of sparse graphs
supervisor: doc. RNDr. Roman Soták, PhD.
study form: full time
Annotation: To study the chromatic invariants of graphs with small number of edges in terms of vertices (linear function). To try to obtain new knowledge for some colorings with respect to maximum average degree.

Generalisation of the Minimum Vertex Cover Problem and the Maximum Independent Set Problem
supervisor: doc. RNDr. Gabriel Semanišin, PhD.
study form: full time
Annotation: The Minimum Vertex Cover Problem and the Maximum Independent Set Problem play a central role in the algorithmic graph theory. Recently a few generalisations of them became important in relation to a communication in various types of networks. The aim of the thesis is to study graph-theoretical and algorithmic aspects of these problems.