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Records in non-life insurance

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Records in non-life insurance

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Abstract

The paper deals with record values as a special case of extremes. A sequence $X_1, X_2, \ldots, X_n, \ldots$ of independent identically distributed random variables with common continuous distribution function F is considered. First we define the basic terms such as record value R_n, number of records N_n among n observations, record time T_n, inter-record time W_n and formulate the well-know limit distribution of R_n, N_n, T_n, W_n. In the next part we derive the general distribution of N_n, which is expressed by recurrence formula. We describe also a simple procedure to calculate the probability of k records among n observations easier. These probabilities we put in a triangle scheme similar the Pascal's triangle. Further we prove that the sequence of record values forms a Markov chain and give the exact distribution of record values in terms of hazard function and the general distribution of the record times and inter-record times . Finally, we apply the theoretical results for real non-life insurance data.

Keywords: record values, record times, inter-record times, triangle scheme, insurance data analysis

Mathematics Subject Classification 2000: 60E05, 62P05

1 Introduction

We can see record events in many fields of real life. For example, well known are sport records, records in meteorology or hydrology, also in financial sphere and insurance. Our main interest will be about records in non-life insurance which have random character. Record claim amounts can ruin the insurance company, so it is very important to follow and forecast such unfavorable random events.

Records are special case of extreme values. The model of record values was introduced by Chandler in 1952 [4], who studied the stochastic behavior of records and defined the basic terms as record value, record time, inter-record time and frequency of records. The first limit theorems about the distribution of these variables belong to Rényi (1962), Neuts (1969), and Resnick (1973) (see [3]). The basic theorem in the theory of records is Resnick's theorem about limit distributions of record values [5]. At the end of twentieth century many papers about the exact distribution of the above mentioned variables in some special cases were published. In 2005 Ahsanullah presented in monograph [1] the recent developments of classical records (records of independent identically distributed random variables) distributed according to exponential law, generalized extreme value distribution, generalized Pareto distribution and power law.

In this paper, we derive general distributions of record values, number of records, record times and inter-record times and calculate some basic characteristics. Further we prove that the sequence of record values forms a Markov chain. At the end we realize real data analysis and compare the limit and exact results.

More precisely, in Section 2 we define the basic terms and formulate the above mentioned limit theorems. In section 3 we derive the general distribution for the total number of records by a recurrence formula. The corresponding probabilities are calculated using a simple procedure and put to a special triangle scheme. In section 4 we give the exact distribution of record values in terms of hazard functions and prove that the sequence of records forms a Markov chain. Section 5 is about the general distribution of record times and inter-record times. Section 6 is devoted to real non-life insurance data analysis.

2 Basic terms and limit results

Consider the sequence $\{X_n, n \geq 1\}$ of independent, identically distributed (iid) random variables with common absolutely continuous distribution function F. Variable X_n is an upper record if $X_n > max\{X_1, X_2, \ldots, X_{n-1}\}$ and lower record if $X_n < min\{X_1, X_2, \ldots, X_{n-1}\}$. Our interest is only in upper records so we call them simple records. By convention X_1 is a record value.

Let $\{T_n, n \geq 1\}$ be the record times at which record values occur. We consider discrete time and define $T_1 = 1$ and $T_n = min\{i, i > T_{n-1}, X_i > X_{T_{n-1}}, n > 1\}$. So the sequence $\{R_n, n \geq 1\} = \{X_{T_n}, n \geq 1\}$ is a sequence of record values. We denote as $Wn = T_{n+1} - T_n$, $n \geq 1$ the inter-record times. The total number of records N_n among n observations $X_1, \ldots, X_n$ is defined by indicators $I_1, \ldots, I_n$ as $N_n = I_1 + \cdots + I_n$ where

$$I_k = \begin{cases} 1, & \text{if } X_k \text{ is a record} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The basic theorem in the theory of record values is the following theorem about the limit distribution of record values proved by Resnick in 1973 [5].

Theorem 2.1. *The class of limit laws for record values is of the form* $\Phi(-\ln(-\ln G(x)))$*, where* $G(x)$ *is standard extreme value distribution and* $\Phi(x)$ *is standard normal distribution. More explicitly, the limit laws must be the type* *of one of the following three distributions:*

$$1)\Phi(x)$$
$$2)\Phi_{1,\alpha}(x) = \begin{cases} 0, & x < 0 \\ \Phi(\ln x^{\alpha}), & x \geq 0 \end{cases}$$
$$3)\Phi_{2,\alpha}(x) = \begin{cases} \Phi(\ln(-x)^{-\alpha}), & x < 0 \\ 1, & x \geq 0, \end{cases}$$

where $\alpha > 0$*.*

The distributions in the Theorem 2.1 are normal, log-normal and negative log-normal and they are derived from Gumbel, Fréchet and Weibull distributions, respectively. These standard extreme value distributions were studied in [6].

The second important limit theorem is the generalized central limit theorem for the total number of records N_n, for record times T_n, and for inter-record times W_n proved by Rényi in 1962, Neuts in 1969, and Resnick in 1973, respectively (see [3]).

Theorem 2.2. *Let* $\{X_n, n \geq 1\}$ *be a sequence of iid random variables with absolutely continuous distribution function* F*. Then for* $n \to \infty$

$$\frac{N_n - \ln n}{\sqrt{\ln n}} \xrightarrow{d} N(0,1), \tag{2}$$
$$\frac{\ln T_n - n}{\sqrt{n}} \xrightarrow{d} N(0,1), \tag{3}$$
$$\frac{\ln W_n - n}{\sqrt{n}} \xrightarrow{d} N(0,1). \tag{4}$$

By the Theorem 2.2 the total number of records N_n increases slowly (as $\ln n$, $n \to \infty$), but the record times and inter-record times increase exponentially. It can be shown (see [2]), that the mean value of the record time is infinite.

3 The total number of records and its distribution

We assume that all variables $X_1, \ldots, X_n$ are each other different with probability 1. Thus they can be ordered in $n!$ ways. Random variable X_n is a record if its value is greater than the other $n-1$ variables, which can be ordered in $(n-1)!$ ways. So the probability that X_n is a record can be expressed as

$$p_n = P(X_n \text{ is a record}) = P(I_n = 1) = \frac{(n-1)!}{n!} = \frac{1}{n}. \tag{5}$$

Now we will concentrate on the probability that among $X_1, \dots, X_n$ we have just k records. We denote this probability by $p_{n,k} = P(N_n = k)$ and derive the recurrence relation for its calculation.

Theorem 3.1. *For each natural number n and for each natural number $k \in \{1, \dots, n\}$ it holds*

$$p_{k,n} = P(N_n = k) = \frac{1}{n} \cdot p_{k-1,n-1} + \frac{n-1}{n} \cdot p_{k,n-1}, \qquad (6)$$

where we put $p_{1,1} = 1$, $p_{k,0} = 0$ and $p_{k,n} = 0$ for $k \notin \{1, \dots, n\}$.

Proof. First we consider the special cases $k = 1$ and $k = n$. One record among $X_1, \dots, X_n$ means that the greatest value is X_1. We can order $X_1, \dots, X_n$ in $n!$ ways. Such orderings where the first value is the greatest is $(n-1)!$, thus $p_{1,n} = \frac{1}{n}$. In the same way we obtain the probabilities $p_{n,n} = \frac{1}{n!}$. Next, if we denote the random events

H_n ... variable X_n is a record,
H_n^c ... variable X_n is not a record,
$A_{i,j}$ ... in the sequence $X_1, \dots, X_j$ are just i records, then

$$p_{k,n} = P(A_{k,n}) = P(A_{k-1,n-1} \cap H_n) + P(A_{k,n-1} \cap H_n^c) \qquad (7)$$

Because the random events $A_{k-1,n-1}$ and H_n (similarly as $A_{k,n-1}$ and H_n^c) are independent, the relation (7) gives us

$$\begin{aligned} p_{k,n} &= P(A_{k-1,n-1}) \cdot P(H_n) + P(A_{k,n-1}) \cdot P(H_n^c) \\ &= p_{k-1,n-1} \cdot P(H_n) + p_{k,n-1} \cdot P(H_n^c). \end{aligned}$$

By (5) we have $P(H_n) = \frac{1}{n}$ and $P(H_n^c) = \frac{n-1}{n}$ so the relation (6) holds. □

To do the calculation of probability by (6) easier we realize the next procedure (in program Maple).

begin
$p_{1,1} := 1;$
for i from 2 to n do
  for k from 1 to i do
    if $k - 1 > 0$ then $a := p_{i-1,k-1}$; $b := p_{i-1,k}$;
      else $a := 0$; $b := p_{i-1,k}$;
    end if;
    $p_{i,k} := \frac{i-1}{i} \cdot b + \frac{1}{i} \cdot a;$
  end do;
end do;
end ;

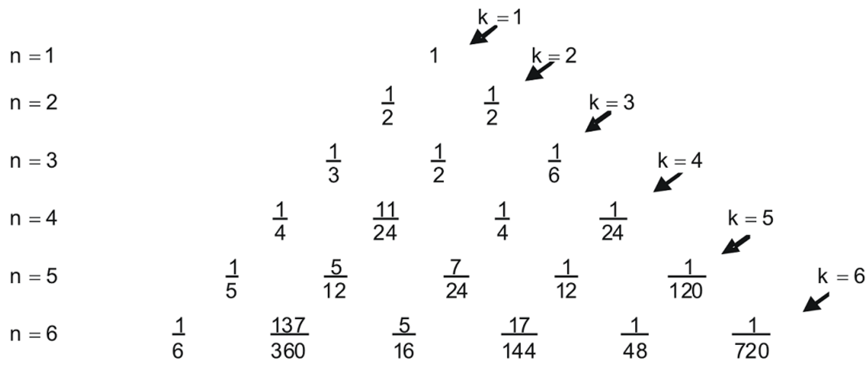

Figure 1: Triangle scheme for the probabilities $p_{k,n}$.

We can draw the probabilities (6) into triangle scheme which is similar like the Pascal's triangle, but we consider here weighted sums of the corresponding values.

The rows represent the total number of observations and the supplementary diagonals the number of records among these observations. For example the value $\frac{11}{24}$ means the probability that among four observations there are two records and it can be obtained by addition $\frac{1}{4} \cdot \frac{1}{3}$ and $\frac{3}{4} \cdot \frac{1}{2}$. If the value in the triangle did not have its left or right component, we put it zero. For instance $\frac{1}{24}$ can be obtained as the sum of $\frac{1}{4} \cdot \frac{1}{6}$ and 0.

Theorem 3.2. *Let $\{X_n, n \geq 1\}$ be a sequence of iid random variables. Then for the expected value of the total number of records and for its variance hold*

$$E(N_n) = \sum_{k=1}^{n} \frac{1}{k} \quad \textit{and} \quad D(N_n) = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k^2} \right). \tag{8}$$

Proof. The proof is simple, based on the independence of the indicators I_n defined by (1) (for the proof of independence see [2]).

$$E(N_n) = \sum_{k=1}^{n} I_k \cdot p_k = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$$

and

$$D(N_n) = D(\sum_{k=1}^{n} I_k) = \sum_{k=1}^{n} D(I_k) = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k^2} \right).$$

□

The sum $1 + \frac{1}{2} + \cdots + \frac{1}{n}$ in (8) can be approximated (see [1]) by

$$E(N_n) \doteq \ln n + \gamma, \ n \to \infty \tag{9}$$

and the variance by

$$D(N_n) \doteq \ln n + \gamma - \frac{\pi^2}{6},\ n \to \infty \tag{10}$$

where $\gamma = 0,5772156649\dots$ is the well known Euler constant. For illustration of this approximation in Table 1 we gives the exact and asymptotic values (denoted as estimation) of $E(N_n)$, $D(N_n)$.

n	$E(N_n)$	$\hat{E}(N_n)$	$D(N_n)$	$\hat{D}(N_n)$
10	2,928968	2,879801	1,379201	1,234867
100	5,187378	5,182386	3,552394	3,537452
1000	7,485471	7,484971	5,841536	5,840037
10000	9,787606	9,787556	8,142772	8,142622

Table 1: Exact and asymptotic values of $E(N_n)$ and $D(N_n)$

4 Distribution of record values

In the first theorem of this part we show that the sequence $\{R_n, n \geq 1\}$ forms a Markov chain. The second theorem give us the general distribution of record values in terms of hazard functions.

Theorem 4.1. *Let $\{X_n, n \geq 1\}$ be a sequence of iid random variables with distribution function F and $\{R_n, n \geq 1\}$ be the corresponding sequence of record values. Then $\{R_n, n \geq 1\}$ forms a Markov chain with conditional probabilities*

$$P(R_{n+1} > x | R_1 = x_1, \dots, R_n = x_n) = \frac{1 - F(x)}{1 - F(x_n)},\ x > x_n. \tag{11}$$

Proof. First we define for arbitrary $n = 1, 2, \dots$ random variables

$$Y_1 = X_{T_n+1},\ Y_2 = X_{T_n+2},\ Y_3 = X_{T_n+3}, \dots\ .$$

This variables are iid with distribution function F and are independent of $R_1, \dots, R_n$. If we denote $z(u)$ the index of the first variable in sequence $Y_1, Y_2, Y_3, \dots$ exceeding value u, then $R_{n+1} = X_{T_n+z(R_n)} = Y_{z(R_n)}$. Now we derive the probability in (11):

$$\begin{aligned}
&P(R_{n+1} > x | R_1 = x_1, ..., R_n = x_n) = P(X_{T_n+z(R_n)} > x | R_1 = x_1, ..., R_n = x_n) \\
&= P(X_{T_n+z(x_n)} > x | R_1 = x_1, ..., R_n = x_n) = P(Y_{z(x_n)} > x | R_1 = x_1, ..., R_n = x_n) \\
&= P(Y_{z(x_n)} > x) \\
&= P(Y_1 > x) + P(Y_1 \leq x_n, Y_2 > x) + ... + P(Y_1 \leq x_n, ..., Y_{k-1} \leq x_n, Y_k > x) + ... \\
&= (1 - F(x)) + F(x_n) \cdot (1 - F(x)) + ... + F^{k-1}(x_n) \cdot (1 - F(x)) + ... \\
&= (1 - F(x)) \cdot \left[1 + F(x_n) + F^2(x_n) + ...\right] = \frac{1 - F(x)}{1 - F(x_n)}.
\end{aligned}$$

We showed that

$$P(R_{n+1} > x | R_1 = x_1, \ldots, R_n = x_n) = P(R_{n+1} > x | R_n = x_n)$$

what means $\{R_n, n \geq 1\}$ is a Markov chain with conditional probabilities (11). $\square$

Definition 4.2. *The function $H(x)$ defined as $H(x) = -\ln(1 - F(x))$ is called hazard function. The derivation of hazard function is called hazard rate and is denoted by $h(x) = H'(x)$.*

Theorem 4.3. *If $F_n(x)$ is the distribution function of random variable R_n, $n \geq 1$ and $H(x)$ is the hazard function according to the distribution function $F(x)$ of $X_n, n \geq 1$, then*

$$F_n(x) = \int_{-\infty}^{x} \frac{H^{n-1}(u)}{(n-1)!} dF(u) \tag{12}$$

and the corresponding density function is

$$f_n(x) = \frac{H^{n-1}(x)}{(n-1)!} f(x). \tag{13}$$

Proof. First we derive (12), (13) for $n = 1, 2, 3$ and then generalize the relation for arbitrary $n \in N$.

$$F_1(x) = P(R_1 \leq x) = P(X_{T_1} \leq x) = P(X_1 \leq x) = F(x)$$

$$F_2(x) = P(R_2 \leq x) = P(X_{T_2} \leq x) = \int_{-\infty}^{y} P(X_{T_2} \leq x | X_{T_1} = u) dF_1(u)$$

$$= \int_{-\infty}^{x} \int_{-\infty}^{y} \sum_{i=1}^{\infty} (P(X_1 \leq u))^{i-1} dF(u) dF(y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \sum_{i=1}^{\infty} F^{i-1}(u) dF(u) dF(y)$$

$$= \int_{-\infty}^{x} \int_{-\infty}^{y} \frac{dF(u)}{1 - F(u)} dF(y) = -\int_{-\infty}^{x} \ln(1 - F(y)) dF(y) = \int_{-\infty}^{x} H(y) dF(y).$$

The probability density of R_2 is given as $f_2(x) = H(x)f(x)$.

$$
\begin{aligned}
F_3(x) &= P(R_3 \le x) = P(X_{T_3} \le x) = \int\limits_{-\infty}^{y} P(X_{T_3} \le x | X_{T_2} = u) dF_2(u) \\
&= \int\limits_{-\infty}^{x} \int\limits_{-\infty}^{y} \sum_{i=1}^{\infty} (P(X_1 \le u))^{i-1} H(u) dF(u) dF(y) \\
&= \int\limits_{-\infty}^{x} \int\limits_{-\infty}^{y} \sum_{i=1}^{\infty} F^{i-1}(u) H(u) dF(u) dF(y) = \int\limits_{-\infty}^{x} \int\limits_{-\infty}^{y} \frac{H(u)}{1 - F(u)} dF(u) dF(y) \\
&= \int\limits_{-\infty}^{x} \frac{H^2(y)}{2!} dF(y),
\end{aligned}
$$

thus the probability density of R_3 is $f_3(x) = \frac{H^2(x)}{2!} f(x)$.
In general, for the distribution function of $R_n, n \ge 1$ we get

$$
\begin{aligned}
F_n(x) &= \int\limits_{-\infty}^{x} dF(u_n) \int\limits_{-\infty}^{u_n} \frac{1}{1 - F(u_{n-1})} dF(u_{n-1}) \cdots \int\limits_{-\infty}^{u_2} \frac{1}{1 - F(u_1)} dF(u_1) \\
&= \int\limits_{-\infty}^{x} \frac{H^{n-1}(y)}{(n-1)!} dF(y),
\end{aligned}
$$

and the corresponding probability density is given by $f_n(x) = \frac{H^{n-1}(x)}{(n-1)!} f(x)$. □

5 Record times and inter-record times

Theorem 5.1. *The distribution of the record times T_r for all $r \ge 2$ and all $k \ge r$ is*

$$
P(T_r = k) = \frac{1}{k} \cdot p_{r-1,k-1}. \tag{14}
$$

Proof.

$$
\begin{aligned}
P(T_r = k) &= P(\text{ the } r\text{-th record occurs in time } k) \\
&= P(X_k \text{ is a record, } X_1, \ldots, X_{k-1} \text{ forms } r-1 \text{ records}) \\
&= P(X_k \text{ is a record }) \cdot P(X_1, \ldots, X_{k-1} \text{ forms } r-1 \text{ records}) \\
&= \frac{1}{k} \cdot p_{r-1,k-1}.
\end{aligned}
$$

□

Theorem 5.2. *The distribution of the inter-record times $W_n, n \geq 1$ is*

$$P(W_n = k) = \sum_{j=0}^{k-1} \binom{k-1}{j} (-1)^j \frac{1}{(j+2)^n}, \; k = 1, 2, \dots \; . \tag{15}$$

Proof. Using the formula of total probability, relation (13) and the binomial theorem, and considering the conditional probability

$$P(W_n = k | X_{T_n} = x_n) = F^{k-1}(x_n) \cdot (1 - F(x_n))$$

we get

$$\begin{aligned} P(W_n = k) &= \int_{-\infty}^{\infty} F^{k-1}(x) \cdot (1 - F(x)) \frac{H^{n-1}(x)}{(n-1)!} dF(x) \\ &= \frac{(-1)^{n-1}}{(n-1)!} \int_0^1 (1-y)^{k-1} y (\ln y)^{n-1} dy \\ &= \frac{(-1)^{n-1}}{(n-1)!} \sum_{j=0}^{k-1} \binom{k-1}{j} (-1)^j \int_0^1 y^{j+1} (\ln y)^{n-1} dy. \end{aligned}$$

We denote the last integral by $I(n)$. To compute $I(n)$ we use a recurrent relation

$$I(n) = \left[\frac{y^{j+2}}{j+2} (\ln y)^{n-1} \right]_0^1 - \frac{n-1}{j+2} \int_0^1 y^{j+1} (\ln y)^{n-2} dy = -\frac{n-1}{j+2} I(n-1)$$

$$I(1) = \frac{1}{j+2},$$

so we get $I(n) = (-1)^{n-1} \frac{(n-1)!}{(j+2)^n}$. Then we can write

$$\begin{aligned} P(W_n = k) &= \frac{(-1)^{n-1}}{(n-1)!} \sum_{j=0}^{k-1} \binom{k-1}{j} (-1)^j (-1)^{n-1} \frac{(n-1)!}{(j+2)^n} \\ &= \sum_{j=0}^{k-1} \binom{k-1}{j} (-1)^j \frac{1}{(j+2)^n}. \end{aligned}$$

□

The Table 2 specifies probabilities (15) for some n and k, where $P_{n,k} = P(W_n = k)$.

$P_{n,k}$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$
$n = 1$	0,50000	0,16667	0,08333	0,05000	0,03333	0,02381	0,01786
$n = 2$	0,25000	0,13889	0,09028	0,06417	0,04833	0,03793	0,03068
$n = 3$	0,12500	0,08796	0,06655	0,05276	0,04323	0,03630	0,03106
$n = 4$	0,06250	0,05015	0,04171	0,03558	0,03093	0,02728	0,02434
$n = 5$	0,03125	0,02713	0,02400	0,02151	0,01950	0,01782	0,01641
$n = 6$	0,01563	0,01425	0,01313	0,01218	0,01137	0,01067	0,01005
$n = 7$	0,00781	0,00736	0,00696	0,00661	0,00630	0,00603	0,00578

Table 2: Distribution law for inter-record times

6 Real data analysis

In this section we realize the practical analysis of non-life insurance data from a Czech insurance company and our main interest is about record values and record times. The set of data consists of 3742 claims in Czech crowns coming from the time interval 3.1.2006 to 4.12.2007 (source [7]). The claims are from the area of property insurance, building insurance and insurance against disasters. In Table 3 the basic characteristics of data are presented.

Count	3 742
Minimum	509
Maximum	13 500 010
Mean	114 252,969
Median	24 413
Variance	$2,78 \cdot 10^{11}$
Skewness	13,759
Kurtosis	247,752
Range	13 499 501

Table 3: Characteristics of data

On the Figure 2 is the time series plot which allows us to identify the record values, record times and the total number of records.

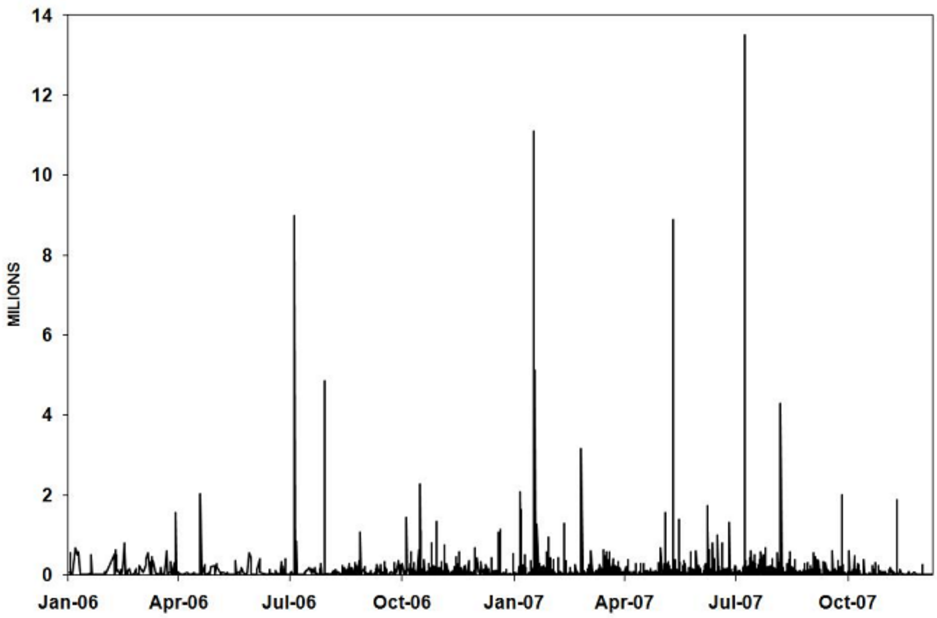

Figure 2: Time series plot

We found 10 record claims in the analyzed time series. So we now know that the variable N_{3742} is equal to 10. The times when records occur and the record values are given in the Table 4.

n	T_n	Date of occurrence	R_n
1	1	3.1.2006	56 800
2	2	4.1.2006	93 801
3	4	4.1.2006	556 381
4	7	8.1.2006	662 345
5	42	17.2.2006	798 733
6	108	31.3.2006	1 567 400
7	122	20.4.2006	2 021 825
8	218	6.7.2006	8 995 000
9	1144	18.1.2007	11 099 861
10	2714	10.7.2007	13 500 010

Table 4: Record times and record values

The graphical representation of the number of records is situated on the Figure 3 and the graphical representation of the record values we can see on the Figure 4.

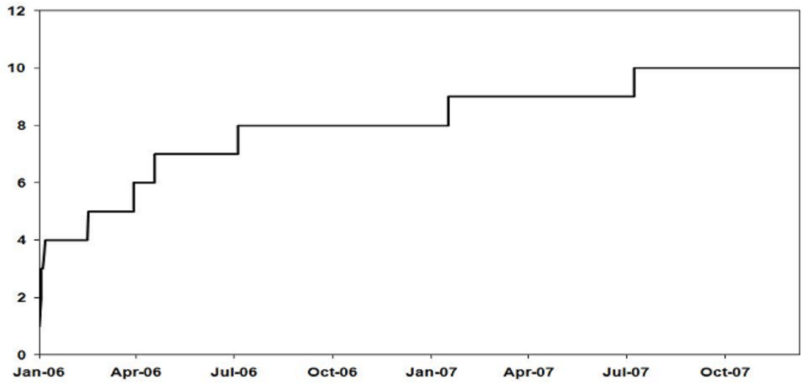

Figure 3: Total number of records

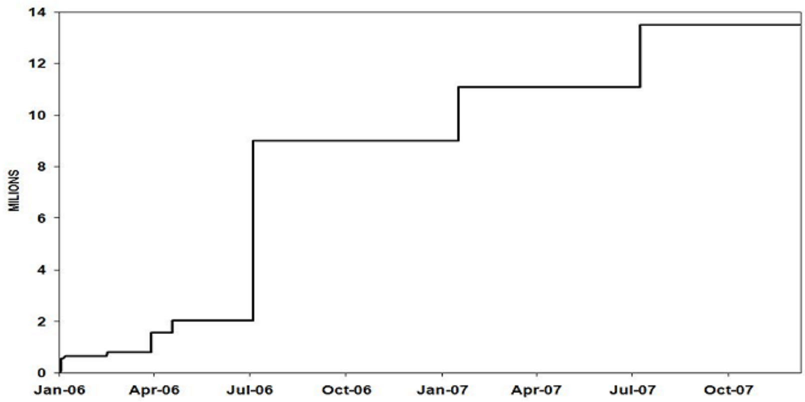

Figure 4: Record values in time

On the Figure 3 we can see slowly increasing number of records in time. Using relations (8), (9) and (10) we calculate the exact and asymptotic values of $E(N_n)$ and $D(N_n)$

$$E(N_n) = 8,80473 \qquad D(N_n) = 7,16006$$
$$\hat{E}(N_n) = 8,80459 \qquad \hat{D}(N_n) = 7,15966.$$

7 Conclusion

In this paper, we presented some theoretical results about record values, as a special case of extremes. Using these results we realized simple statistical analysis of real data. Work on detailed analysis of actual data is in progress. We considered only classical records when the observations are independent identically distributed random variables. Further research would be required to describe the possible dependence of records and to characterize special types of record distributions.

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References

[1] AHSANULLAH, M. : Record values - theory and application. University Press of America, Inc.,Lanham, Maryland, USA, 2005. ISBN 0-7618-2794-3.

[2] ANDĚL, J.: Matematika náhody. Matfyzpress, Praha, 2000. ISBN 80-85863-52-9.

[3] GULATI S., PADGETT W. J.: Parametric and nonparametric inference from record-breaking data. Springer-Verlag, New York 2003, ISBN 0-387-00138-7

[4] CHANDLER, K. N. : The distribution and frequency of record values. Journal of the Royal Statistical Society. Series B (Methodological), Vol. 14, No. 2 (1952), pp. 220-228

[5] RESNICK, S. I. : Limit laws for record values. Stochastic Processes and their Applications 1 (1973) 67-82. North-Holland Publ. CO.

[6] SKŘIVÁNKOVÁ, V., TARTAĽOVÁ, A.: Catastrophic risk management in non-life insurance. E+M Economics and Management, 2/2008, 65-72. ISSN 1212-3609.

[7] ZIMANOVÁ, J.: Štatistická analýza extrémnych hodnôt. Diplomová práca na PF UPJŠ, Košice, 2008.

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