P. J. ŠAFÁRIK UNIVERSITY
FACULTY OF SCIENCE
INSTITUTE OF MATHEMATICS
Jesenná 5, 040 01 Košice, Slovakia

G. Karafová and R. Soták

Generalized fractional total coloring of complete graphs for sparse edge properties

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Generalized fractional total coloring of complete graphs for sparse edge properties

Gabriela Karafová[1] and Roman Soták[2]

Institute of Mathematics
Faculty of Science, P. J. Šafárik University
Jesenná 5, 040 01 Košice, Slovakia

e-mail: [1]*gabriela.karafova@student.upjs.sk*
[2]*roman.sotak@upjs.sk*

Abstract: An additive and hereditary property of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphism. Let $\mathcal{P}$ and $\mathcal{Q}$ be two additive and hereditary graph properties and let r, s be integers such that $r \geq 2s$. Then $\frac{r}{s}$-*fractional* $(\mathcal{P}, \mathcal{Q})$-*total coloring* of a finite graph G is a mapping $f : V \cup E \to \binom{\{1,2,\ldots,r\}}{s}$ such that for any color i all vertices of color i induce a subgraph from the property $\mathcal{P}$, all edges of color i induce a subgraph from the property $\mathcal{Q}$ and vertices and incident edges have assigned disjoint sets of colors. The minimum value of the ratio $\frac{r}{s}$-fractional $(\mathcal{P}, \mathcal{Q})$-total coloring of G is called *fractional* $(\mathcal{P}, \mathcal{Q})$-*total chromatic number* $\chi''_{f,\mathcal{P},\mathcal{Q}}(G) = \frac{r}{s}$. Let $k = \sup\{i : K_{i+1} \in \mathcal{P}\}$. Let $zn + y,\ y, z \in \mathbb{R}$ be maximum number of edges that form a graph on n vertices from the property $\mathcal{Q}$. We show for an arbitrary additive and hereditary property $\mathcal{P}$ and for some properties $\mathcal{Q}$ of certain types, that $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{k+1}$ or $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n(n+(2z-1))}{2(zn+y)}$ depending on n, k and $\mathcal{Q}$ for $n > k + 1$ and $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n(n-1)}{2(zn+y)} + 1$ for $n \leq k + 1$.

1 Introduction

New results contained in this paper are based on [2], where linear programming methods are used to study generalized fractional total coloring of complete graphs. The most important result in [2] is that we can determine the fractional $(\mathcal{P}, \mathcal{Q})$-total chromatic number of a complete graph as a solution of a linear program with only two inequalities (see e.g. linear program (2)).

We denote the class of all finite simple graphs by $\mathcal{I}$. A *graph property* $\mathcal{P}$ is a non-empty isomorphism-closed subclass of $\mathcal{I}$. A property P is called *additive* if

$G \cup H \in \mathcal{P}$ whenever $G \in \mathcal{P}$ and $H \in \mathcal{P}$. A property $\mathcal{P}$ is called *hereditary* if $G \in \mathcal{P}$ and $H \subseteq G$ implies $H \in \mathcal{P}$.

We use the following standard notations for specific hereditary properties, which we will use in this paper.

$$\mathcal{O} = \{G \in \mathcal{I} : E(G) = \emptyset\},$$

$$\mathcal{D}_k = \{G \in \mathcal{I} : \text{each subgraph of } G \text{ contains a vertex of degree at most } k\},$$

$$\mathcal{S}_k = \{G \in \mathcal{I} : \Delta(G) \leq k\},$$

$$\mathcal{T} = \{G \in \mathcal{I} : G \text{ is a planar graph}\},$$

where $\Delta(G)$ is the maximum degree of the graph G.

Borowiecki and Mihók [1] studied graph properties and showed that the set of all additive and hereditary properties forms a complete distributive lattice $(\mathbb{L}^a, \subseteq)$, with $\mathcal{O}$ being its smallest element and $\mathcal{I}$ its greatest one.

A *total coloring* of a graph G is a coloring of the vertices and edges (together the *elements* of G) such that all adjacent and incident elements obtain distinct colors. The minimum number of colors of a total coloring of G is called total chromatic number $\chi''(G)$ of G.

($\mathcal{P}$,$\mathcal{Q}$)-independent set is a subset of $V \cup E$ such that the vertices in this set induce a graph from the property $\mathcal{P}$, edges induce a graph from the property $\mathcal{Q}$ and moreover vertices and edges are not incident.

Definition 1. *Let $I_1, I_2, \ldots, I_t,\ \ t \in \mathbb{N}$ be all (maximal) ($\mathcal{P}$,$\mathcal{Q}$)-independent sets in G. A fractional ($\mathcal{P}$,$\mathcal{Q}$)-total coloring of G is a mapping g, which assigns to each set $I_j,\ \ j = 1, \ldots, t$ a non-negative weight $g(I_j)$ such that $\sum_{I_j \ni u} g(I_j) \geq 1$ for each element $u \in V \cup E$. The fractional ($\mathcal{P}$,$\mathcal{Q}$)-total chromatic number $\chi''_{f,\mathcal{P},\mathcal{Q}}(G)$ of G is the least total weight of the fractional ($\mathcal{P}$,$\mathcal{Q}$)-total coloring of G.*

Note that Definition 1 corresponds to the following linear program:

$$\sum_{j=1}^{t} g(I_j) \to \min;$$

$$\sum_{I_j \ni u} g(I_j) \geq 1, \quad \forall u \in V \cup E; \tag{1}$$

$$g(I_j) \geq 0, \quad \forall j = 1, \ldots, t.$$

Now let $\mathcal{P} \supseteq \mathcal{O}$ and $\mathcal{Q} \supseteq \mathcal{O}_1$ be two additive and hereditary graph properties and $G = (V, E)$ be a graph on n vertices. Let $k = c(\mathcal{P}) = \sup\{i : K_{i+1} \in \mathcal{P}\}$ be the *completness* of a property $\mathcal{P}$. Let the maximum number of edges, which can be colored by the same color as is used on i vertices, be denoted by a_i. We consider a_i as a function of $n-i$ and the property $\mathcal{Q}$: $a_i = a(n-i, Q)$. It is easy to see that the sequence $\{a_i\}_{i=0}^{k+1}$ is decreasing.

In the following we will often use only notation a_i. According to [2] $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n)$ can be found as an optimal solution of the following linear program.

$$\begin{aligned} \sum_{i=0}^{k+1} x'_i &\to \min; \\ \sum_{i=0}^{k+1} i x'_i &\geq n; \\ \sum_{i=0}^{k+1} a_i x'_i &\geq \frac{n(n-1)}{2}; \\ x'_i \geq 0, &\quad \forall i = 0, \dots, k+1. \end{aligned} \tag{2}$$

2 $\chi''_{\mathbf{f},\mathcal{P},\mathcal{Q}}(\mathbf{K_n})$ for the linear form of $\mathbf{a}(\mathbf{n}-\mathbf{i}, \mathbf{Q})$

In this section we deal with such graph properties $\mathcal{Q}$ corresponding to the edges of complete graphs, that the aforementioned parameter a_i has a linear form and is non-negative. In this case the fractional $(\mathcal{P}, \mathcal{Q})$-total coloring of a complete graph on n vertices depends on $n, c(\mathcal{P})$ and a_0.

Theorem 1. *Let $\mathcal{P}$ and $\mathcal{Q} \neq \mathcal{O}$ be two additive and hereditary graph properties, $n \in \mathbb{N}$, $n > c(\mathcal{P}) + 1$ and let $a_i = a(n-i, Q)$ has a linear form $a_i = \max\{z(n-i) + y, 0\}$, where $y, z \in \mathbb{R}$ for each $i = 0, 1, \dots, c(\mathcal{P}) + 1$. For $2z - c(\mathcal{P}) - 1 \neq 0$ denote $t := 1 + \frac{2zc(\mathcal{P}) - 2y}{2z - c(\mathcal{P}) - 1}$. Then:*

1) $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{c(\mathcal{P})+1}$, if:

- *1a) $2z - c(\mathcal{P}) - 1 > 0$ and $n \geq t$*
- *1b) $2z - c(\mathcal{P}) - 1 = 0$ and $y \geq zc(\mathcal{P})$*
- *1c) $2z - c(\mathcal{P}) - 1 < 0$ and $n \leq t$*

2) $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n(n+(2z-1))}{2(zn+y)}$, if:

- *2a) $2z - c(\mathcal{P}) - 1 > 0$ and $n < t$*
- *2b) $2z - c(\mathcal{P}) - 1 = 0$ and $y < zc(\mathcal{P})$*
- *2c) $2z - c(\mathcal{P}) - 1 < 0$ and $n > t$*

If $y < 0$ then cases 1b) and 1c) cannot occur.

Proof. Denote $c(\mathcal{P}) = k$. Consider $\mathcal{Q} = \mathcal{O}$, $n > c(\mathcal{P}) + 1$ and $a_i = \max\{z(n-i)+y, 0\}$, where $y, z \in \mathbb{R}$. Then $a_i \geq 0$ for every $i = 0, 1, \dots, k+1$. First, we show that $z \geq 0$. It is easy to see that

$$\forall i = 0, 1, \dots, k : a_i \geq a_{i+1}$$

$$z(n-i) + y \geq z(n-i-1) + y$$

$$z \geq 0$$

Note that if $z = 0$ then also $y = 0$ and it means that $\mathcal{Q} = \mathcal{O}$ but we do not consider this case. Thus $z > 0$ and also $a_0 > 0$.

The first pivot in the first row of the next simplex table is $-(k+1)$:

	x'_0	x'_1	x'_2	$\dots$	x'_{k-1}	x'_k	x'_{k+1}	p_1	p_2	
	1	1	1	$\dots$	1	1	1	0	0	0
p_1	0	-1	-2	$\dots$	$-(k-1)$	$-k$	$-(k+1)$	1	0	$-n$
p_2	$-a_0$	$-a_1$	$-a_2$	$\dots$	$-a_{k-1}$	$-a_k$	$-a_{k+1}$	0	1	$-\frac{n(n-1)}{2}$

We get the following table:

	x'_0	x'_1	$\dots$	x'_k	x'_{k+1}	p_1	p_2	
	1	$\frac{k}{k+1}$	$\dots$	$\frac{1}{k+1}$	0	$\frac{1}{k+1}$	0	$-\frac{n}{k+1}$
x'_{k+1}	0	$\frac{1}{k+1}$	$\dots$	$\frac{k}{k+1}$	1	$-\frac{1}{k+1}$	0	$\frac{n}{k+1} \geq 0$
p_2	$-a_0$	$\frac{a_{k+1}}{k+1} - a_1$	$\dots$	$\frac{ka_{k+1}}{k+1} - a_k$	0	$-\frac{a_{k+1}}{k+1}$	1	$\frac{na_{k+1}}{k+1} - \frac{n(n-1)}{2}$

This table is optimal, when $\frac{na_{k+1}}{k+1} - \frac{n(n-1)}{2} \geq 0$. It is equivalent to the expression:

$$n(2z-k-1) \geq 2z-k-1+2zk-2y.$$

Here we have to consider the following cases:

a) 1a) If $2z-k-1 > 0$ and $n \geq \frac{2z-k-1+2zk-2y}{2z-k-1} = t$ then we have optimal solution $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{c(\mathcal{P})+1}$.

2a) If $2z-k-1 > 0$ and $n < \frac{2z-k-1+2zk-2y}{2z-k-1} = t$ then we have to continue in solving according to a second pivot.

b) 1b) If $2z-k-1 = 0$ and $y \geq zk$ then for each integer n we have optimal solution $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n}{c(\mathcal{P})+1}$.

2b) If $2z-k-1=0$ and $y<zk$ then for each integer n we have to continue in solving.

c) 1c) If $2z-k-1<0$ and $n \leq \frac{2z-k-1+2zk-2y}{2z-k-1}=t$ then we have optimal solution $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n)=\frac{n}{c(\mathcal{P})+1}$.

2c) If $2z-k-1<0$ and $n > \frac{2z-k-1+2zk-2y}{2z-k-1}=t$ then we have to continue in solving.

Now we determine a second pivot in the second row of the previous table. Pivot must be negative. Let $k'=\min\limits_{0\leq i\leq k}\{i; a_i\geq 0\}$

$$\max_{0\leq i<k'}\left\{\frac{\frac{k+1-i}{k+1}}{\frac{ia_{k+1}}{k+1}-a_i}\right\}=\max_{0\leq i<k'}\left\{\frac{k+1-i}{i(z(n-(k+1))+y)-(k+1)(z(n-i)+y)}\right\}=$$

$$=\max_{0\leq i<k'}\left\{\frac{k+1-i}{(k+1-i)(-zn-y)}\right\}=\max_{0\leq i<k'}\left\{\frac{1}{-a_0}\right\}=\frac{1}{-a_0}.$$

The element in the second row and column for p_1 can not be a pivot, because

$$a_0\geq a_{k+1} \quad\Leftrightarrow\quad -\frac{1}{a_0}\geq\frac{\frac{1}{k+1}}{-\frac{a_{k+1}}{k+1}}$$

If $a_{k+1}=0$ then we cannot consider $-\frac{a_{k+1}}{k+1}$ as a pivot, because pivot has to be negative. Thus arbitrary element in the second row of the table except values in columns x'_{k+1}, p_1 and p_2 can be considered as the second pivot. Without loss of generality let it be $-a_0$, because it is easier to work with this one. So we get the following table, where

$$\frac{n^2(2z-k-1)+n(2y+(k+1)(1-2z))}{-2(k+1)(zn+y)}\geq 0$$

in each case, in which we do not have any optimal solution after first step of the simplex method.

	x'_0	x'_1	$\ldots$	x'_k	x'_{k+1}	p_1	p_2	
	0	0	$\ldots$	0	0	$\frac{z}{zn+y}$	$\frac{1}{zn+y}$	$-\frac{n(n+2z-1)}{2(zn+y)}$
x'_{k+1}	0	$\frac{1}{k+1}$	$\ldots$	$\frac{k}{k+1}$	1	$-\frac{1}{k+1}$	0	$\frac{n}{k+1}\geq 0$
x'_0	1	$\frac{k}{k+1}$	$\ldots$	$\frac{1}{k+1}$	0	$\frac{zn-zk-z+y}{(k+1)(zn+y)}$	$-\frac{1}{zn+y}$	$\frac{n^2(2z-k-1)+n(2y+(k+1)(1-2z))}{-2(k+1)(zn+y)}$

Here we can see that optimal solution in these cases is:

$$\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n)=\frac{n(n+(2z-1))}{2(zn+y)}.$$

Now let $y<0$.

1b) In this case it can not hold that $y \geq zk$, because $a_i \geq 0$ for each $i = 0, 1, \ldots, k+1$ and $z > 0$.

1c) We know that $t = \frac{2z-k-1+2zk-2y}{2z-k-1} = 1 + \frac{2zk-2y}{2z-k-1}$. The fraction $\frac{2zk-2y}{2z-k-1}$ has negative denominator and positive numerator for $y < 0$. Thus $t < 1$, so there is no positive integer $n \leq t$.

□

In the case $n = 1$ trivially $\chi''_{f,\mathcal{P},\mathcal{Q}}(K_1) = 1$. The following theorem is about results for $2 \leq n \leq c(\mathcal{P}) + 1$.

Theorem 2. *Let $\mathcal{P}$ and $\mathcal{Q} \neq \mathcal{O}$ be two additive and hereditary graph properties, $n \in \mathbb{N}, 2 \leq n \leq c(\mathcal{P}) + 1$ and let $a_i = a(n-i, Q)$ has a linear form $a_i = \max\{z(n-i) + y, 0\}$, where $y, z \in \mathbb{R}$ for each $i = 0, 1, \ldots, c(\mathcal{P}) + 1$. Then*

$$\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n(n-1)}{2(zn+y)} + 1.$$

Proof. First, we show that $y \leq 0$. It is easy to see that now we have variables $x'_0, x'_1, \ldots, x'_n$ and $a_n = 0$, $a_{n-1} = 0$ and $a_{n-2} = 1$. Therefore $y = 1 - 2z$. According to the previous proof we know that $z \geq 0$ and it follows that $y \leq -1$. If $z = 0$ then $y = 1$ and it means that every a_i is constant, what is not interesting for us. Now we solve the following linear problem:

	x'_0	x'_1	$\ldots$	x'_{n-3}	x'_{n-2}	x'_{n-1}	x'_n	p_1	p_2	
	1	1	$\ldots$	1	1	1	1	0	0	0
p_1	0	-1	$\ldots$	$-(n-3)$	$-(n-2)$	$-(n-1)$	$-n$	1	0	$-n$
p_2	$-a_0$	$-a_1$	$\ldots$	$-a_{n-3}$	-1	0	0	0	1	$-\frac{n(n-1)}{2}$

As a first pivot in the second row of the table can be considered $-a_0$, because

$$\max_{0 \leq i \leq n-2} \left\{ -\frac{1}{a_i} \right\} = -\frac{1}{a_0}.$$

We get the table:

	x'_0	x'_1	$\ldots$	x'_{n-3}	x'_{n-2}	x'_{n-1}	x'_n	p_1	p_2	
	0	$1 - \frac{a_1}{a_0}$	$\ldots$	$1 - \frac{a_{n-3}}{a_0}$	$1 - \frac{1}{a_0}$	1	1	0	$\frac{1}{a_0}$	$-\frac{n(n-1)}{2(zn+y)}$
p_1	0	-1	$\ldots$	$-(n-3)$	$-(n-2)$	$-(n-1)$	$-n$	1	0	$-n$
x'_0	1	$\frac{a_1}{a_0}$	$\ldots$	$\frac{a_{n-3}}{a_0}$	$\frac{1}{a_0}$	0	0	0	$-\frac{1}{a_0}$	$\frac{n(n-1)}{2(zn+y)}$

We need to find the second pivot, because we do not have an optimal solution, yet:

$$\max\left\{\max_{1\le i\le n-2}\left\{\frac{1-\frac{a_i}{a_0}}{-i}\right\}, \frac{1}{-(n-1)}, \frac{1}{-n}\right\} = \max\left\{\frac{z}{-(zn+y)}, \frac{1}{-(n-1)}, \frac{1}{-n}\right\} = \frac{1}{-n}$$

Therefore, the second pivot in the first row is $-n$ and we get the last table:

	x_0'	x_1'	$\dots$	x_{n-2}'	x_{n-1}'	x_n'	p_1	p_2	
	0	$1-\frac{a_1}{a_0}-\frac{1}{n}$	$\dots$	$1-\frac{1}{a_0}-\frac{n-2}{n}$	$1-\frac{n-1}{n}$	0	0	$\frac{1}{a_0}$	$-\frac{n(n-1)}{2(zn+y)}-1$
x_n'	0	$\frac{1}{n}$	$\dots$	$\frac{n-2}{n}$	$\frac{n-1}{n}$	1	$-\frac{1}{n}$	0	1
x_0'	1	$\frac{a_1}{a_0}$	$\dots$	$\frac{1}{a_0}$	0	0	0	$-\frac{1}{a_0}$	$\frac{n(n-1)}{2(zn+y)}$

The elements $1-\frac{a_i}{a_0}-\frac{i}{n}\ge 0$ for each $i=1,\dots,n$ because $y\le 0$. Also the elements in the last column and two last rows are non-zero. It means that we have an optimal solution

$$\chi''_{f,\mathcal{P},\mathcal{Q}}(K_n) = \frac{n(n-1)}{2(zn+y)} + 1.$$

□

The following two claims are easy corollaries of Theorems 1 and 2. It is well known that for ℓ-degenerate graphs the number of edges is bounded by a linear function of n. More precisely, $a_i=\ell(n-i)-\frac{\ell(\ell+1)}{2}$, which can be proved routinely by induction.

Corollary 1. *Let $\mathcal{P}$ be additive and hereditary property and $\mathcal{Q}=\mathcal{D}_\ell$. Then*

$$\chi''_{f,\mathcal{P},\mathcal{D}_\ell}(K_n) = \begin{cases} \frac{n}{c(\mathcal{P})+1}, & \text{if} \quad c(\mathcal{P}) < 2\ell-1 \ \&\ n \ge 1+\frac{2\ell c(\mathcal{P})+\ell(\ell+1)}{2\ell-c(\mathcal{P})-1}, \\ \frac{n(n-1)}{2\ell n-\ell(\ell+1)}+1, & \text{if} \quad n\le c(\mathcal{P})+1, \\ \frac{n(n+2\ell-1)}{2\ell n-\ell(\ell+1)}, & \text{otherwise.} \end{cases}$$

Corollary 2. *Let $\mathcal{P}$ be additive and hereditary property and $\mathcal{Q}=\mathcal{T}$. Then*

$$\chi''_{f,\mathcal{P},\mathcal{T}}(K_n) = \begin{cases} \frac{n}{c(\mathcal{P})+1}, & \text{if} \quad c(\mathcal{P}) < 5 \ \&\ n \ge \frac{5c(\mathcal{P})+17}{5-c(\mathcal{P})}, \\ \frac{n(n-1)}{2(3n-6)}+1, & \text{if} \quad n\le c(\mathcal{P})+1, \\ \frac{n(n+5)}{2(3n-6)}, & \text{otherwise.} \end{cases}$$

Proof. Let $k = c(\mathcal{P})$. We have $a_i = 3(n-i) - 6$ for each $i = 0, \ldots, k+1$, because graph property $\mathcal{T}$ is a class of planar graphs and so $z = 3$, $y = -6$ and $t = \frac{5k+17}{5-k}$. □

Note that $\chi''_{f,\mathcal{P},\mathcal{T}}(K_n) = \chi''_{f,\mathcal{P},\mathcal{D}_3}(K_n)$ because a_i is the same in these both cases.

If $\mathcal{Q} = \mathcal{S}_l$ then $a_i = \frac{l(n-i)}{2}$ and it has a linear form for $n - i \geq l + 1$. Therefore if $n \geq l + c(\mathcal{P}) + 2$ then we can use Theorems 1 and 2 for this case, too.

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