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Igor Fabrici and Roman Soták, eds.

Workshop
Mikro Graph Theory

March 19 – 23, 2012 Herľany

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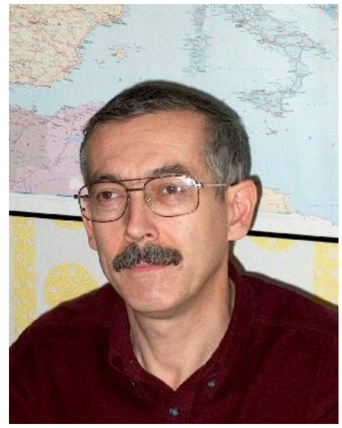
Sixty years of Professor Mirko Horňák



Professor Mirko Horňák was born on March 22th, 1952 in Topoľčany. He spent the years of early childhood in Bánovce nad Bebravou, the city where worked his parents teachers at secondary school. In 1958, after the family moved to Košice, he entered the elementary school at Hviezdoslavova Street, and two years later, he entered the elementary school with extended education of mathematics (now the school at Park Angelinum). His interest in sciences fully developed at secondary school at Kováčska 28, where he participated in Olympiad in mathematics, physics and chemistry (winning several national rounds of these competitions); besides these activities, he was also a keen athlete and basketball player. He never doubted about his future studies the mathematics was the indisputable choice.

In 1970, he started his university studies in pure mathematics at the Faculty of Sciences of P.J. Šafárik University in Košice. After the graduation in 1975, he was employed at the mathematics department at his alma mater, working there until today; in 1988, he was designated for associate professor and in 2011 (after several years of blockade from the Ministry of Education despite of the university approval for the professorship), he became a full professor. At the Department of geometry and Algebra (now the Institute of Mathematics), he was teaching various subjects ranging from discrete mathematics to numerical mathematics, queuing theory, theory of matroids and codes.

His scientific career evolved under the guidance of Professor Ernest Jucovič, one of pioneers of Slovak combinatorics. The range of research topics he participated on is wide: it includes the structure of planar and embedded graphs (unavoidable configurations of plane graphs, regular and near-regular cell decompositions of 2-manifolds), contributions in chromatic graph theory (cyclic, achromatic, neighbour-distinguishing and on-line proper colourings, graph observability), decompositions of graphs into trails, and the structure of hypergraphs. The areas of his interest are commonly regarded as difficult and dehortative, yet the results he obtained are far from being easy; a notable trademark of his proofs is very high level of mathematical accuracy and rigorousness, the fact that positively influences many of his students and colleagues. He is the author of more than 45 papers with more than 120 citations, and the supervisor of five PhD. theses. On several occasions, he visited important research institutions (Banach Center in Poland, EHESS in Paris, Keio University in Japan) establishing a prolific mutual cooperation with researchers from abroad. For twenty years, he was the member of the organizing committee of the graph theory workshop Cycles and Colourings. He was the guest editor of several special volumes of Discrete Mathematics journal which were devoted to C&C workshop, and he is a member of editorial board of Discussiones Mathematicae Graph Theory and Opuscula Mathematica journals.

Outside of mathematics, Mirko Horňák is known as keen contract bridge player (often high scoring in east-Slovakian bridge tournaments) and a dedicated tourist liking mushroom-picking. His hidden face is best revealed in a circle of colleagues and friends when singing classic Czech, Slovak or international hit songs of his youth. And, those participants of the Cycles and Colourings workshop who recurrently come back to High Tatras, always enjoy his video-presentations from last year conference; some of them maybe start dreaming on another extreme mountain trip (usually nontrivial walk with many unforgettable views) guided by Mirko!

In agreement with Pablo Picasso that One starts to get young at the age of sixty and then it is too late, we wish Mirko to live - being young in heart - many happy years and a lot of joy within and outside of mathematics.

Katarína Horňáková
Tomáš Madaras
Roman Soták

Cycles through 4 - connected Mikro sets of planar graphs - A generalization of Tutte's Theorem

Jochen Harant

In 1956, W.T. Tutte proved that a 4-connected planar graph is hamiltonian. Moreover, in 1997, D.P. Sanders extended this to the result that a 4-connected planar graph has a hamiltonian cycle containing any two of its edges. We prove that a planar graph G has a cycle through a given subset X of its vertex set and through any two prescribed edges of the subgraph of G induced by X if $|X| \geq 3$ and if X is 4-connected in G. If $X = V(G)$ then Sanders' result follows.

We also discuss the problem under which condition a 4-connected planar graph has a hamiltonian cycle through more than two specified edges.

Cyclic words and vertex colourings of plane graphs

Stanislav Jendrol'

Let $\mathbb{A} = \{a, b, c, \dots\}$ be a finite alphabet, whose element are called letters (digits, colours, symbols, ...).

The *word* of length n over $\mathbb{A}$ is an expression $w = a_1 a_2 \dots a_n$, where $a_i \in \mathbb{A}$ for all $i = 1, 2, \dots, n$. *Subword* $\bar{w}$ of the word w is an expression $\bar{w} = a_i a_{i+1} \dots a_j$ with $1 \leq i \leq j \leq n$.

The *cyclic word* of length n is an expression $w = a_1 a_2 \dots a_n$, $n \geq 2$ (Consider the cyclic word as a sequence of consecutive labells on the vertices of a cycle of length). A subword of a cyclic word is its arbitrary part.

Let us recall some properties that words can have. A word is *proper* if it no two consecutive letters are the same. For example the word "*abcba*" is proper but the word "*abbcd*" is not proper.

The word $a_1 a_2 \dots a_n$, $n \geq 1$, is simple if $a_i \neq a_j$ for $i \neq j$.

The word of the form $a_1 a_2 \dots a_{2k}$ with property that $a_i = a_{i+k}$ for all $i = 1, 2, \dots, k$ is called the *repetition*. A word is called *nonrepetitive* is none of its subwords is a repetition.

Examples: The word "*abcabc*" is a repetition, the word "*abcacbd*" is nonrepetitive, while the word "*abcbcdea*" is not nonrepetitive because it contains a subword "*bcbc*", which is a repetition.

A *palindrom* is any word which can be read in the same way from the front and from the back. The word is *palindromfree* if no its subword is a palindrom

Examples: The words "*abcddcba*" and "*abcdcba*" are palindroms while the word *abcdbda* is palindrom free.

A word is a *weak parity* one if at least one letter in it appears there an odd number of times. A word is a *strong parity* one if each used letter in it is used there an odd numbers of times. For example the word "*abcabde*" is a weak parity word. The word "*abcabdadbd*" is a strong parity word.

Consider a 2-connected plane graphs. All its faces are bounded by cycles, called the *facial cycles*. If we label all vertices of a 2-connected plane graph G with letters from an alphabet $\mathbb{A}$, then any face $\alpha = [v_1, v_2, \ldots, v_k]$ determined by the vertices $v_1, v_2, \ldots, v_k$ can be associated with a cyclic word $a_1 a_2 \ldots a_k$, where k is size (degree) of the face α and a_i is a labell of the vertex v_i. The word $a_1 a_2 \ldots a_k$ is called the facial word of the face α of G.

In our talk we will consider the following problem:

Problem: What is the minimum number of letters in an alphabet $\mathbb{A}$ that allows to label the vertices of a given 2-connected plane graph G in such a way that all the facial words of G over $\mathbb{A}$ have a given property $\mathcal{P}$?

We will give a survey on results and open questions concerning this problem for several properties of words.

Rainbow Cycles in Cube Graphs

Arnfried Kemnitz

(joint work with Jens-P. Bode)

A graph G is called rainbow with respect to an edge coloring if no two edges of G have the same color. Given a host graph H and a guest graph $G \subseteq H$, an edge coloring of H is called G-anti-Ramsey if no subgraph of H isomorphic to G is rainbow. The anti-Ramsey number $f(H, G)$ is the maximum number of colors for which there is a G-anti-Ramsey edge coloring of H. We consider cube graphs Q_n as host graphs and cycles C_k as guest graphs and present some general bounds for $f(Q_n, C_k)$ as well as the exact values for $n \leq 4$.

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Rainbow numbers and rainbow connection

Ingo Schiermeyer

In this talk we consider edge colourings of graphs. A subgraph H of a graph G is called rainbow subgraph, if all its edges are coloured distinct. In the first part we will survey the computation of rainbow numbers. For given graphs G,H the rainbow number $rb(G, H)$ is the smallest number m of colours such that if we colour the edges of G with at least m different colours, then there is always a totally multicoloured or rainbow copy of H. For various graph classes of H we will list the known rainbow numbers if G is the complete graph and report about recent progress on the computation of rainbow numbers. Finally, new results on the rainbow numbers $rb(Q_n, Q_k)$ for the hypercube Q_n will be presented.

An edge-coloured graph G is called rainbow connected if any two vertices are connected by a path whose edges have distinct colours. This concept of rainbow connection in graphs was introduced by Chartrand et al.. The rainbow connection number of a connected graph G, denoted $rc(G)$, is the smallest number of colours that are needed in order to make G rainbow connected. In the second part we will show several complexity results and present lower and upper bounds for $rc(G)$. We will discuss $rc(G)$ for graphs with given minimum degree. Finally, we will present some recent Erdős-Gallai type results for $rc(G)$.

On a P_k-free colouring

Gabriel Semanišin

We study a special case of Vertex Deletion Problem: Find a minimum weight set of vertices of a given graph whose deletion gives a graph satisfying a given property. A subset S of vertices of a graph G is called a *k-path vertex cover* if the graph $G \setminus S$ is P_k-free. The minimum cardinality of a k-path vertex cover in G is denoted $\psi_k(G)$. We show that the problem of determining $\psi_k(G)$ is NP-hard for each $k \geq 2$, while for trees the problem can be solved in linear time. We investigate upper bounds on the value of $\psi_k(G)$ and provide several estimations and exact values of $\psi_k(G)$. We also prove that $\psi_3(G) \leq (2n + m)/6$, for every graph G with n vertices and m edges. Moreover, we provide some exact and approximation algorithms for determining the value of ψ_k for some classes of graphs.

Wilson-Schreiber Colourings of Cubic Graphs

Martin Škoviera

An $\mathcal{S}$-colouring of a cubic graph G is an edge-colouring of G by points of a Steiner triple system $\mathcal{S}$ such that the colours of any three edges meeting at a vertex form a block of $\mathcal{S}$. In this note we present an infinite family of point-intransitive Steiner triple systems $\mathcal{S}$ such that (1) every simple cubic graph is $\mathcal{S}$-colourable and (2) no proper subsystem of $\mathcal{S}$ has the same property. Only one point-intransitive system satisfying (1) and (2) was previously known. This is a joint work with M. Grannell, T. Griggs, and E. Mačajová.

On the Thue characteristics of graphs

Erika Škrabuľáková

(joint work with Jens Schreyer)

A sequence $r_1, r_2, \ldots, r_{2n}$ such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$ is called a *repetition*. A sequence is called *non-repetitive* if no of its subsequences forms a repetition. Let G be a graph whose vertices are coloured. A colouring φ of the graph G is *non-repetitive* if the sequence of colours on any path in G is non-repetitive. The *Thue chromatic number*, denoted by $\pi(G)$, is the minimum number of colours of a non-repetitive colouring of G. Moreover, if the colour of every vertex v is chosen only from a list $L(v)$ of colours assigned to the vertex v we speak about a *non-repetitive list colouring* φ_L of the graph G with list assignment L. If the graph G is non-repetitively list colourable for every list assignment L with list size at least k, we call G *non-repetitively k-choosable*. The smallest number k such that G is non-repetitively k-choosable is called the *Thue choice number* of G and is denoted by $\pi_{ch}(G)$.

Here we compare the Thue chromatic number and the Thue choice number of graphs. We show that there exist examples of infinite families of graphs where these parameters give the same value. On the other hand we give to our knowledge the first example of an infinite family of graphs (Gummi-bear graphs) where $\pi(G) < \pi_{ch}(G)$.

Generalized Pell sequences, their interpretations and matrix generators

Andrzej Włoch

In this talk we generalize the companion Pell sequence. We give combinatorial, graph and matrix representations of this sequence. Using these representations we describe some properties of the generalized Pell numbers and the generalized companion Pell numbers. We define the golden Pell matrix for determining the generalized Pell sequences and among other we prove the "generalized Cassini formula" for them.

On edge-colourings, colour sets and distinguishing of vertices of a graph

Mariusz Woźniak

Let G be a finite simple graph and let $f : E(G) \to C$ be an edge colouring of G.

The *colour set* of a vertex $v \in V(G)$ (with respect to f) is the set $S(v)$ of colours of edges incident with v. The colouring f *distinguishes* two vertices if, for instance, their colour sets are distinct. How many colours we need to be able to distinguish (for instance) all vertices?

There are many modifications of the above mentioned problem. We shall discuss some of them.

Adjacent vertex distinguishing edge colorings of graphs

Norma Zagaglia Salvi

Let $G = (V, E)$ be a finite, simple, undirected graph. A *proper edge coloring* of G is a map α from E to a set of colors C such that adjacent edges have different colors. The *chromatic index* $\chi'(G)$ of a graph G is the minimum number of colors in a proper edge coloring of G. By *Vizing's theorem*, we have that the chromatic index of any graph G satisfies the bounds $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of G. The color set of a vertex $v \in V$ is the set $C_\alpha(v)$, or simply $C(v)$, of colors assigned to the edges incident to v. A proper edge coloring of G is *adjacent vertex distinguishing* (*avd* for short) if $C(v) \neq C(w)$ whenever the vertices v and w are adjacent [1, 3]. Such a coloring was also called *adjacent strong edge* [20] and *neighbour-distinguishing* [16]. The complete graph K_2 on two vertices does not admit an *avd*-coloring and more generally, this is true for any graph having K_2 as a connected component. So we can consider only connected graphs on at least three vertices.

The *avd chromatic index* $\chi'_a(G)$ of G is the minimum number of colors in an *avd*-coloring of G. Clearly, if there exists an *avd*-coloring of G, then $\chi'_a(G) \geq \chi'(G)$ and $\chi'_a(G) \geq \Delta(G)$. Moreover, if G has two adjacent vertices of degree $\Delta(G)$, then $\chi'_a(G) \geq \Delta(G) + 1$.
In [20] the authors determined the *avd* chromatic indices for paths, cycles, trees, complete graphs and complete bipartite graphs. They also conjectured that an analogous of Vizing's theorem also holds for the *avd* chromatic index.

Conjecture 1 *If G is a simple connected graph on at least 3 vertices and different from the cycle C_5, then*

$$\Delta(G) \leq \chi'_a(G) \leq \Delta(G) + 2\,.$$

In [1] the conjecture was confirmed for all bipartite graphs and for graphs with $\Delta = 3$, while in [13] the author, using a probabilistic method, was able to proof that every graph G with $\Delta > 10^{20}$ has $\chi'_a(G) \leq \Delta + 300$. In [6] the conjecture was proved for planar graphs of girth at least 6. In [8] the authors determined the adjacent vertex-distinguishing chromatic indices of the hypercubes.
In [12] it is defined the general neighbour-distinguishing index of a graph G which consists in the minimum k in a general (non proper) k-edge coloring of G which is neighbour-distinguising; this parameter is denote $gndi(G)$. In [16] the authors proved that if $\chi(G) \geq 3$, then the minimum number of colors in a general neighbour-distinguishing coloring, denoted $gndi(G)$, satisfies $\lceil log_2 \chi(G) \rceil + 1 \leq gndi(G) \leq \lceil log_2 \chi(G) \rceil + 2$.
Moreover if $log_2 \chi(G) \notin Z$, then $gndi(G) = \lceil log_2 \chi(G) \rceil + 1$.
The *direct product* $G \times H$ of two graphs $G = (V, E)$ and $H = (W, F)$ is the graph with vertex-set $V(G \times H) = V \times W$ and edge-set $E(G \times H) =$

$\{(av, bw) : ab \in E,\ vw \in F\}$. This product is commutative and associative (up to isomorphisms). The direct product of a bipartite graph by any other graph is bipartite. So, the direct product of any graph G by a path or a cycle is always bipartite, except when the cycle has odd length and G is not bipartite. As usual, C_n, P_n and K_n denote a cycle, a path and a complete graph on n vertices, respectively. Clearly, $P_2 = K_2$ and $C_3 = K_3$.

In [11] it is proved that the *avd* chromatic index of the direct product of a d-regular graph G by a path of length at least 3 is given by

$$\chi'_a(G \times P_m) = \begin{cases} 2d & \text{for } m = 3 \\ 2d+1 & \text{for } m > 3\,. \end{cases}$$

Moreover it is proved that if G is a d-regular graph, where $d > 2$, for a positive integer $m > 4$, when even, or $m \geq 2d+1$, when odd, then $\chi'_a(G \times C_m) = 2d+1$.

In this talk we survey recent results about the adjacent vertex distinguishing chromatic index of a graph. In particular we focus on a graph which is the direct product of a graph by a path or a cycle or a graph with a loop in every vertex. in [17] we prove that for a graph G and a path P_m with $m \geq 4$

$$2\Delta(G) \leq \chi'_a(G \times P_m) \leq 2\Delta(G) + 1.$$

Moreover, we establish some conditions under which the *avd* chromatic index coincides with the ordinary chromatic index. Finally, we obtain some results concerning the *avd* chromatic index of the generalized Petersen graphs.

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Programme of the Workshop

Monday		
16:00 - 22:00	Registration	
18:00 - 20:00	Dinner	

Tuesday		
08:00 - 09:00	Breakfast	
09:30 - 10:30	Woźniak M.	On edge-colourings, colour sets and distinguishing of vertices of a graph
10:30 - 11:00	Coffee break	
11:00 - 12:00	Zagaglia Salvi N.	Adjacent vertex distinguishing edge colourings of graphs
12:30 - 13:30	Lunch	
afternoon	Problem solving (individually)	
18:30 - 19:30	Dinner	

Wednesday		
08:00 - 09:00	Breakfast	
09:30 - 10:30	Harant J.	Cycles through 4 - connected Mikro sets of planar graphs - A generalization of Tutte's Theorem
10:30 - 11:00	Coffee break	
11:00 - 11:20	Škoviera M.	Wilson-Schreiber colouring of cubic graphs
11:30 - 11:50	Włoch A.	Generalized Pell sequences, their interpretations and matrix generators
12:00 - 13:00	Lunch	
13:30 - 17:30	Trip - Rankovské skaly	
18:30 - 19:30	Dinner	

Thursday		
08:00 - 09:00	Breakfast	
09:30 - 10:30	Schiermeyer I.	Rainbow numbers and rainbow connection
10:30 - 11:00	Coffee break	
11:00 - 11:20	Kemnitz A.	Rainbow cycles in cube graphs
11:30 - 11:50	Škrabuľáková E.	On the Thue characteristics of graphs
12:30 - 13:30	Lunch	
15:30 - 16:30	Jendroľ S.	Cyclic words and vertex colourings of plane graphs
16:30 - 17:00	Coffee break	
17:00 - 17:20	Semanišin G.	On a P_k-free colouring
18:30 -	Bithday party	

Friday		
07:30 - 08:30	Breakfast	
08:30 - 09:30	Bača M.	Connection between α-labelling and antimagic labellings
12:00 - 13:00	Lunch	