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PRACTICAL PLACEMENT OF TRAINEE TEACHERS TO SCHOOLS

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Abstract. Several countries successfully use centralized matching schemes for assigning students to study places or fresh graduates to their first positions. In this paper we explore the computational aspects of a possible similar scheme for assigning trainee teachers to schools. The special feature of this model is that each teacher specializes in two subjects that have to be performed in the same school. We show that the model becomes intractable already under several strict restrictions concerning the total number of subjects and the number of acceptable schools each teacher is allowed to list.

Keywords: assignment of students, bipartite matching, algorithm, NP-completeness

1 Introduction

The traditional study of teachers-to-be in Slovakia involves the specialization of each student in two subjects, e.g. Mathematics and Physics, Chemistry and Biology, Slovak language and English etc. In addition to the study of the various topics of these subjects, principles of Pedagogics and Psychology, each curriculum contains a practical placement in a real school several times during the study. Students might try to find suitable schools by themselves, but to ensure the quality of such a placement, the faculties require that in each school a student is supervised by a qualified and experienced teacher who is approved by the faculty for taking this responsibility. Hence it is often the case that the faculty provides a list of such schools and the students may choose from the list.

The assignment is often performed on the first-come-first-served basis. However, not all schools provide supervisors for all subjects, or they may not have enough classes to accept several students for a particular subject. This might be a serious problem, as a student is usually required to follow both his/her subjects in the same school (even if each subject is supervised by a different teacher, placement at two different schools might be infeasible for example because of the school time table and time loss needed for travelling). So it might happen that for some unhappy students no place remains, or they might be forced to go to a school that is located neither in the town of their residence nor of the faculty, thus increasing their costs above the acceptable level.

The aim of this paper is to study the computational complexity of the trainee teachers matching problem. For several special cases we propose efficient algorithms that allocate all applicants to acceptable schools or decide that such an allocation is impossible: if there are altogether only 2 specialization subjects, or there are 3 subjects but each school can accept at most 1 student for each subject (irrespectively of her other specialization), or, without the restriction on the number of specialization subjects, if each applicant is allowed to list at most two acceptable schools and each school has at most one place for each specialization. By contrast, we show that the problem to decide whether a full assignment exists is NP-complete if there are 3 subjects and schools may have capacity 2 in one of its subjects, or if there are 4 subjects and each school has capacity at most 1 in each subject.

2 Related work

The classical problems of combinatorial optimization like the maximum cardinality bipartite matching problem, assignment problem, or flow problem have successfully been applied to various variants of manpower allocation problems (see e.g. applications reviewed [3], Chapter 12). Practical situations have lead also to some NP-complete variants [9]. Recently, a lot of attention has been attracted by several large-scale centralized allocation schemes used for assigning pupils to public schools in Boston and New York [1], [2], assigning graduates of medical schools to their first jobs in hospitals in the USA [13], [14], university applicants to study places in Hungary [5] etc. In such schemes, the applicants as well as schools, in addition to simply stating acceptability, are also required to order the other side of the 'market' according to their preferences. For an overview of other applications, various models and their computational complexity, the reader is adviced to consult the recently published monograph by David Manlove [12] or the comprehensive web page containing a description of matching practices for various levels of education in many European countries [15].

Of the models studied so far the closest to our situation are the so called hospital-residents problem with couples: members of a married couple wish to go to a pair of geographically close hospitals, see [8] or even refuse to be separated and insist on going to the same institution [11]. Another case is the Scottish scheme for medical students that have to be assigned to two training units (medical and surgical one), however, these two assignments have to be allocated to two differents semesters [10]. Our model differs from all ones presented so far due to the applicants specialization, the necessity to perform both subjects in the same school and schools allowed to have different capacities for different subjects.

3 Definitions

An instance J of the Teachers Assignment problem, TAP for short, involves a set A of applicants, a set S of schools and a set P of subjects. For ease of exposition, elements of the set P will sometimes be referred to by letters like M, F, I or B, to remind of real subjects taught at schools, like Mathematics, Physics, Informatics or Biology etc.

Each applicant $a \in A$ is characterized by a pair of different subjects $\mathbf{p}(a) = \{p_1(a), p_2(a)\} \subseteq P$. Sometimes we shall also say that a particular applicant is of type MF, MB, or IB, etc.

Each school $s \in S$ has a certain capacity for each subject, the vector of capacities will be $\mathbf{c}(s) = (c_1(s), \dots, c_{|P|}(s)) \in \mathbb{N}^{|P|}$, an entry of $\mathbf{c}(s)$ will also be referred to as a *partial capacity* of school s. Here, $c_p(s)$ is the maximum number of students whose specialization involves subject p that school s is able to accept. Again, we shall sometimes write $c_M(s), c_I(s)$ etc.

A school s is compatible with applicant a if $c_p(s) \geq 1$ for both subjects $p \in \mathbf{p}(a)$. We suppose that each applicant a provides a list $S(a)$ of *acceptable* schools, i.e. schools to which he/she willing to go. An *assignment* $\mathcal{M}$ is a subset of $A \times S$ such that each applicant $a \in A$ is a member of at most one pair in $\mathcal{M}$. We shall write $\mathcal{M}(a) = s$ if $(a, s) \in \mathcal{M}$ and say that applicant a is *assigned* (to school s); if there is no such school, applicant a is *unassigned*. The set of applicants assigned to a school s will be denoted by $\mathcal{M}(s) = \{a \in A; (a, s) \in \mathcal{M}\}$. We shall also denote by $\mathcal{M}_p(s)$ the set of applicants assigned to s whose specialization includes subject p and by $\mathcal{M}_{p,r}(s)$ the set of applicants assigned to s whose specialization is exactly the pair $\{p, r\}$. More precisely,

$$\mathcal{M}_p(s) = \{a \in A; (a, s) \in \mathcal{M} \ \& \ p \in \mathbf{p}(a)\}$$

and

$$\mathcal{M}_{p,r}(s) = \{a \in A; (a, s) \in \mathcal{M} \ \& \ \{p, r\} = \mathbf{p}(a)\}.$$

An assignment $\mathcal{M}$ is *feasible* if $\mathcal{M}(a) \in S(a)$ for each $a \in A$ and $|\mathcal{M}_p(s)| \leq c_p(s)$ for each school s and each subject p.

Example. Suppose there are 3 subjects M,F and I, four applicants a_1 of type IF, a_2 of type MF and a_3, a_4 of type MI. There are two schools s_1, s_2 with $c_M(s_1) = 1$, $c_F(s_1) = c_I(s_1) = 2$ and $c_M(s_2) = 2$, $c_F(s_2) = c_I(s_2) = 1$. Both schools are acceptable for all applicants.

Here it is possible to assign all applicants, namely $\mathcal{M}(a_1) = \mathcal{M}(a_3) = s_1$ and $\mathcal{M}(a_2) = \mathcal{M}(a_4) = s_2$. However, suppose that applicant a_1 arrives first and he/she chooses s_2. Then the only option for a_2 is to go to school s_1, but then no place remains for applicants a_3 and a_4.

This shows that in situations when all applicants could get a place, an unsuitable order of arrivals may leave half of them unassigned.

FULL-TAP denotes the problem to decide, given an instance J of TAP, whether a full feasible assignment exists, i.e. such that leaves no students unassigned. In the following section we explore the computational complexity of several special cases of FULL-TAP.

4 Computational complexity

Theorem 1 FULL-TAP *is solvable in polynomial time in each of the following cases:*

(i) $|P| = 2$*;*

(ii) $|P| = 3$ *and no partial capacity of a school exceeds* 1*;*

(iii) $|P|$ *is arbitrary, but each applicant is allowed to list at most two acceptable schools and all partial capacities are at most 1.*

Proof. For case (i) it suffices to realize that all applicants are essentially equivalent and a school with partial capacities c_1 and c_2 can admit at most $c = \min\{c_1, c_2\}$ students. Hence FULL-TAP reduces to the classical bipartite b-matching problem that can be solved in polynomial time by any well-known algorithm [3].

Similarly, in case (ii) each school can admit at most one applicant, so FULL-TAP is equivalent to the simple maximum cardinality bipartite matching problem, again solvable in polynomial time.

In case (iii) let us proceed in the following way. In the first phase we deal with applicants that list an incompatible school or a school that does not have enough capacity for both specialization subjects. Such schools can be removed from their lists. If we get some applicants with empty lists, FULL-TAP is clearly insolvable. Otherwise, if the list of an applicant contains only one school (let us call these applicants *spoiled*), to get a full assignment, he/she must be assigned to that particular school. This, however, decreases the respective partial capacities of the school involved and new spoiled applicants can emerge. If, in this first phase we are not able to place all spoiled applicants, no full matching exists; otherwise we continue with the second phase with the partial capacities reduced accordingly. (It is easy to see that the first phase can be performed in polynomial time.)

The obtained *cannonical* FULL-TAP instance J has $|S(a)| = 2$ for each $a \in A$. Let us denote $S(a_i) = \{s_i^1, s_i^2\}$ and introduce a boolean variable x_i for each applicant a_i with the following interpretation: if x_i is TRUE, we shall say that a_i is assigned to school s_i^1; if x_i is FALSE, we say that a_i is assigned to school s_i^2. Now create a boolean formula $B(J)$ in the following way. For each pair of applicants a_i, a_j whose specialization involves at least one common subject and for each school $s \in S(a_i) \cap S(a_j)$ we create a clause $C_{i,j,s}$ as follows:

- if $s = s_i^1$ and $s = s_j^1$ then $C_{i,j,s} = \bar{x}_i + \bar{x}_j$;
- if $s = s_i^1$ and $s = s_j^2$ then $C_{i,j,s} = \bar{x}_i + x_j$;
- if $s = s_i^2$ and $s = s_j^1$ then $C_{i,j,s} = x_i + \bar{x}_j$;
- if $s = s_i^2$ and $s = s_j^2$ then $C_{i,j,s} = x_i + x_j$.

Clause $C_{i,j,s}$ ensures that a_i and a_j do not both occupy the only place for their common subject at school s. Formula $B(J)$ is then the conjuction of clauses $C_{i,j,s}$ for all triples a_i, a_j, s as described above. It is easy to see that $B(J)$ is solvable if and only if a full assignment for J exists (remember, we assume that J is cannonical). $B(J)$ is a boolean formula in conjunctive normal form and since each clause contains just two literals, its satisfiability can be decided in polynomial time. This concludes that case (*iii*) is also polynomially solvable. ■

Let us remark here that the computational complexity of the case with acceptable sets of cardinality 2 but with arbitrary partial capacities of schools is still open.

In the following theorem we shall use as the starting known NP-complete problem 3-dimensional matching, 3DM in brief, see [7], problem SP1. An instance of 3DM contains three disjoint sets U, V and W, all of cardinality n, and a set of triples $\mathcal{T} \subseteq U \times V \times W$. The question is whether there exists a perfect matching, i.e. a subset $\mathcal{N} \subseteq \mathcal{T}$ such that $|\mathcal{N}| = n$ and $\mathcal{N}$ covers all elements of $U \cup V \cup W$. We shall use the NP-complete restriction of 3DM to such instances where no element occurs in more than 3 triples in $\mathcal{T}$.

Theorem 2 FULL-TAP *is NP-complete already when* $|S(a)| \leq 3$ *and*

(*i*) $|P| = 3$ *and no partial capacity of a school exceeds* 2*; or*

(*ii*) $|P| = 4$ *and no partial capacity of a school exceeds* 1.

Proof. For case (*i*), given an instance $J = (U, V, W, \mathcal{T})$ of 3DM, we construct an instance J' of TAP with 3 subjects (say M,F and I) and $c_M(s) = 2$, $c_F(s) = c_I(s) = 1$ for each school.

For each triple $t \in \mathcal{T}$ we create a school s_t. For each $z \in U \cup V \cup W$ let $\mathcal{T}_z$ be the set of triples in $\mathcal{T}$ containing z and $\ell_z = |\mathcal{T}_z|$. For each $u \in U$ we create applicants $a_u^1, a_u^2, \ldots, a_u^{\ell_u - 1}$, each of type IF; their set will be denoted by A_u. For each $v \in V$ we create an applicant a_v of type MI and for each $w \in W$ an applicant a_w of type MF. For each applicant corresponding to an element $z \in U \cup V \cup W$, acceptable schools are those that correspond to triples in $\mathcal{T}_z$.

Suppose that the 3DM instance J has a perfect matching $\mathcal{N} \subseteq \mathcal{T}$. We assign each applicant in J' to an acceptable school so that the capacity of no school in no subject will be exceeded.

For each $t = (u, v, w) \in \mathcal{N}$ we assign to school s_t applicants a_v and a_w. For each $u \in U$ there are $\ell_u - 1$ triples $t \in \mathcal{T} \setminus \mathcal{N}$ containing u, so to the corresponding schools we assign applicants $a_u^1, a_u^2, \ldots, a_u^{\ell_u - 1}$. It is easy to see that each applicant is assigned to an acceptable school and that the defined assignment obeys all capacities.

Conversely suppose that there exists a full feasible assignment $\mathcal{M}$. Let $S_{\mathcal{N}}$ be the set of schools to which two applicants are assigned in $\mathcal{M}$ and let $\mathcal{N} \subseteq \mathcal{T}$ be the set of corresponding triples. By the construction, if $s_t \in S_{\mathcal{N}}$ and $t = (u, v, w)$ then the assigned applicants are a_v and a_w. Clearly, for two different schools in $S_{\mathcal{N}}$ these two applicants are different and so also any two different triples in $\mathcal{N}$ differ in their elements from V and W. It remains to show that if $t, t' \in \mathcal{N}$ are different then their corresponding elements from U are also different.

To get a contradiction, suppose that some element $u \in U$ belongs to at least two different triples $t, t' \in \mathcal{N}$. Notice that the only acceptable schools for $\ell_u - 1$ applicants of the set A_u are the ℓ_u schools s_t for $t \in \mathcal{T}_u$. If two different schools $s_t, s_{t'}$ belong to $S_{\mathcal{N}}$ then the number of schools that have enough capacity for

$\ell_u - 1$ applicants in A_u and are acceptable for them is at most $\ell_u - 2$. This is a contradiction with the assumption that $\mathcal{M}$ is a full assignment.

The proof for case (i) can easily be modified for (ii) by making the following changes:

- The set of subjects is M,F,I,B,
- each school s has $c_M(s) = c_F(s) = c_I(s) = c_B(s) = 1$;
- for each $v \in V$ the type of applicant a_v is MF;
- for each $w \in W$ the type of applicant a_v is IB;
- for each $u \in U$ contained in ℓ_u triples in $\mathcal{T}$ there are $\ell_u - 1$ applicants of type MI and $\ell_u - 1$ applicants of type FB.

The acceptability is defined in the same way according to the structure of $\mathcal{T}$ and the rest of the proof is analogical. ■

5 Conclusions and open questions

In the quest for a possible centralized matching scheme the presented intractability results are pessimistic. Still, some other computational techniques could be employed, e.g. integer programming formulations. One should also see whether the complexity status of the problem changes if the students are not allowed to express acceptability, i.e. if each student were required to go to any school that provides both subjects of his/her specialization and has a free place for each.

The existing extensive literature on matchings and many succesfull existing schemes call for exploring other possible approaches. One can imagine that students, in addition to expressing acceptability, could be allowed to list the acceptable schools in order of their preference and/or the schools might also be given the right to order students. Then some other criteria for the obtained matching might be considered: Pareto optimality (from the viewpoint of students, see [4]) or stability (introduced by Gale and Shapley [6]).

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