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Approximation algorithms for the teachers assignment problem*

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Abstract

We deal with assigning trainee teachers to schools for a practical placement. The starting point is the situation characteristic for Slovak and Czech education system where each pre-service teacher specializes in two subjects. It is known that when each schools has a certain upper limit for the number of assignees whose specialization involves a given subject, the problem of assigning the maximum number of trainee teachers is NP-hard. In this paper we propose several approximation algorithms for this problem.

Keywords: assignment problem; approximation

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1 Introduction

The traditional study of teachers-to-be in Slovakia and Czech republic involves specialization of each student in two subjects, e.g. Mathematics and Physics, Chemistry and Biology, Slovak language and English etc. Each curriculum of teachers' study contains also several practical placements in a real school. During such placements students visit classes and also teach pupils themselves, always under supervision of an experienced and qualified teacher approved by the university for taking this responsibility. Students might try to find suitable schools and supervising teachers by themselves, but to ensure the quality of such a placement, each faculty usually provides a list of such schools and teachers, and students are assigned to them by the faculty staff.

Finding an acceptable placement of students is not easy and it usually takes several days and many iterations. The aim is to find a place for each student. However, even if the number of approved supervising teachers is sufficient for the current number of students to be placed, this is not always possible, as the structure of available places might not be suitable, not all schools provide supervisors for all subjects, or they may not have enough classes to accept several students for a particular subject, or some students cannot be placed to some schools because of, say, time-consuming commuting.

The classical problems of combinatorial optimization like the maximum cardinality bipartite matching problem, assignment problem, or flow problem have been successfully applied to a range of variants of manpower allocation problems, see e.g. applications reviewed in [1], Chapter 12. Requirements brought about by concrete practical applications can also lead to some NP-complete cases, like the one described in [7]. However, we are not aware of any previous publications that could resemble the teachers assignment problem except for [2, 3] and [4]. There we showed that if the curriculum requires that each student teaches both subjects at one school during the same placement, then the problem to find a matching for the maximum possible number of students is polynomial only if there are altogether only two specialization subjects, or there are three subjects but each school can accept at most one student for each subject (irrespectively of her other specialization). Interestingly, if for each student at most two schools are acceptable and all partial capacities are at most one, a polynomial algorithm exits to decide whether all students can be matched, but the problem of matching the maximum number of students is intractable. The existence of a complete assignment is also NP-complete if

- (i) there are only three subjects and schools may have capacity two in one of their subjects, or if
- (ii) there are four subjects and each school has capacity at most one in each subject.

Moreover, this problem is also NP-complete if each school is acceptable for each student, but now without the restriction on the number of subjects.

We proposed an integer linear program for the teachers assignment problem in [3] and successfully applied it to real data. However, from the theoretical point of view, further research for NP-hard problems is necessary. In this paper we explore the possibilities of polynomial approximation algorithms in this context. In Section 4 we propose some other possible research directions.

2 Definitions and notation

An instance J of the Teachers Assignment problem, TAP for short, involves a set A of n applicants (students, trainee teachers), a set S of m schools and a set $P = \{1, 2, \ldots, k\}$ of k subjects, like Mathematics, Physics, Informatics or Biology etc.

Each applicant $a \in A$ is characterized by a pair of different subjects $\mathbf{p}(a) = \{p_1(a), p_2(a)\} \subseteq P$. The set of applicants whose specialization involves a subject $p \in P$ will be denoted by A_p and the set of those whose specialization is exactly $\{p, r\}$ by $A_{p,r}$. We also suppose that each applicant a provides a list $S(a)$ of *acceptable* schools, i.e. schools to which she (we shall refer to the applicants as females) is willing to go.

Each school $s \in S$ has a certain capacity for each subject, the vector of capacities of school s will be $\mathbf{c}(s) = (c_1(s), \ldots, c_{|P|}(s)) \in \mathbb{N}^{|P|}$. An entry of $\mathbf{c}(s)$ is a *partial capacity* of school s. Here, $c_p(s)$ is the maximum number of applicants from A_p that school s is able to accept.

An *assignment* $\mathcal{M}$ is a subset of $A \times S$ such that each applicant $a \in A$ is a member of at most one pair in $\mathcal{M}$. We shall write $\mathcal{M}(a) = s$ if $(a, s) \in \mathcal{M}$ and say that applicant a is *assigned* (to school s); if there is no such school, applicant a is *unassigned*. The set of applicants assigned to a school s will be denoted by $\mathcal{M}(s) = \{a \in A; (a, s) \in \mathcal{M}\}$. If $\mathcal{M}$ is any assignment, we denote by $\mathcal{M}_p$ and by$\mathcal{M}_{p,r}$ its restrictions to applicants from the sets A_p and $A_{p,r}$, respectively. More precisely,

$$\mathcal{M}_p(s) = \{a \in A; (a, s) \in \mathcal{M} \ \& \ p \in \mathbf{p}(a)\}$$

and

$$\mathcal{M}_{p,r}(s) = \{a \in A; (a, s) \in \mathcal{M} \ \& \ \{p, r\} = \mathbf{p}(a)\}.$$

An assignment $\mathcal{M}$ is a *matching* if $\mathcal{M}(a) \in S(a)$ for each $a \in A$ and $|\mathcal{M}_p(s)| \leq c_p(s)$ for each school s and each subject p.

A matching is *maximal* if no school can accept an unassigned applicant without violating at least one of its partial capacities. A maximal matching can be found quite easily, for example by the following procedure: take each applicant in turn and assign her to any acceptable school that has still enough capacity for both her subjects. A matching is *maximum* if it assigns the maximum number of applicants.

Example 1. Clearly, not each maximal matching is maximum. In the TAP instance with the set of subjects $P = \{1, 2, 3\}$, given in Figure 1, the maximum matching is $\mathcal{M}_1 = \{(a_1, s_1), (a_2, s_1), (a_3, s_2)\}$ of size 3. However, matching $\mathcal{M}_2 = \{(a_3, s_1)\}$ is maximal and its size is 1. This shows that the size of a maximal matching can be only one third of a maximum matching. Theorem 1 (proved below) says that it cannot be less.

school	capacities for			applicant	type	acceptable schools
	1	2	3			
s_1	2	1	1	a_1	$\{1,2\}$	s_1
s_2	0	1	1	a_2	$\{1,3\}$	s_1
				a_3	$\{2,3\}$	s_1, s_2

Figure 1: Instance J for Example 1.

Theorem 1 *If $\mathcal{M}$ is any maximal matching and $\mathcal{M}^*$ is a maximum matching then $|\mathcal{M}| \geq |\mathcal{M}^*|/3$.*

Proof. Suppose that we have a maximum matching $\mathcal{M}^*$ in an arbitrary instance J of TAP and we want to insert into J the pairs matched by a maximal matching $\mathcal{M}$ one by one. If $(a,s) \in \mathcal{M}$ belongs to $\mathcal{M}^*$ too, we do nothing. But, if $(a,s) \notin \mathcal{M}^*$, then the pairs matched by $\mathcal{M}^*$ may prevent (a,s) from being inserted. To create enough place for this pair, we might need to drop from $\mathcal{M}^*$ at most one pair for each subject of a's specialization, plus the pair (a,s') if $(a,s') \in \mathcal{M}^*$ and $s \neq s'$. Hence, $|\mathcal{M}| \geq \frac{1}{3}|\mathcal{M}^*|$. ■

3 Approximation algorithms

MAX-TAP denotes the problem to find a matching with maximum cardinality for a given instance J of TAP. The special case of MAX-TAP where the number of subjects is fixed to k will be denoted by MAX-k-TAP.

Trivially, MAX-2-TAP and MAX-3-TAP with all partial capacities bounded by 1 are polynomially solvable. In the former case all the applicants are equivalent and each school s can accept at most $\min\{c_1(s), c_2(s)\}$ applicants. In the latter case, each school can accept at most one applicant. So both problems reduce to the simple bipartite maximum cardinality (b-)matching problem (for polynomial algorithms see e.g. [9]). By contrast, MAX-3-TAP is NP-complete even if all partial capacities are bounded by 2 and MAX-4-TAP is NP-complete even when all partial capacities are at most 1 (see [3]) .

In this section we study approximation algorithms for MAX-TAP. Theorem 1 gives a trivial 3-approximation algorithm and Example 1 shows that this approximation bound is tight. Below we propose three approximation algorithms with better approximation guarantees. The first two of them are based on finding a maximum cardinality matching in an instance of MAX-TAP for a subset of applicants A_p where subject p is fixed.

First we describe how to find such a matching by employing network flow methods. For an instance J of TAP and a fixed subject $p \in P$ we create a network $N(J_p)$. Its vertices are: a vertex v_a for each applicant $a \in A_p$, a vertex $u_{s,r}$ for each school and for each subject r such that $c_r(s) > 0$, plus two vertices σ and τ (source and sink). The arcs of $N(J_p)$ and their capacities are given in Figure 2.

(σ, v_a)	capacity 1	for all $a \in A_p$
$(v_a, u_{s,r})$	capacity 1	for all $s \in S(a)$, all subjects $r \neq p$ and for all $a \in A_{p,r}$
$(u_{s,r}, u_{s,p})$	capacity $c_r(s)$	for all $s \in S$ and all subjects $r \neq p$
$(u_{s,p}, \tau)$	capacity $c_p(s)$	for all $s \in S$

Figure 2: Arcs and their capacities for the flow network $N(J_p)$.

As all capacities are integral, by [5] there exists an integer maximum flow f that can be used to define a feasible matching as follows: for an applicant $a \in A_{p,r}$ we set $\mathcal{M}(a) = s$ if $f(v_a, u_{s,r}) = 1$. It is easy to see that $\mathcal{M}$ is indeed a matching: each applicant is assigned to an acceptable school and no partial capacity is exceeded.

The number of vertices in $N(J_p)$ is bounded by $n + mp + 2$, the upper bound for the number of its arcs is $n + nm + 2mp + 2$. The push-relabel algorithm with dynamic trees uses $O(NM\ log(N^2/M))$ operations for a network with N vertices and M arcs [6]. This means that for an instance of MAX-TAP there are at most k iterations that need to solve such a flow problem, hence the whole algorithm can be accomplished in time that is polynomial in the number of applicants, schools and subjects.

3.1 Algorithm Greedy1

Let us consider now the algorithm depicted in Figure 3.

begin fix the order of subjects $1, 2, \ldots, k$;
for $p := 1$ to k **do**
  begin find a maximum cardinality matching $\mathcal{M}^p$ for A_p;
    reduce the partial capacities of schools accordingly
  end
end

Figure 3: Algorithm Greedy1

Theorem 2 *The approximation bound of algorithm* Greedy1 *is 2.*

Proof. We prove by induction on the number k of subjects. For $k = 2$, Greedy1 clearly finds a maximum size matching, so the theorem holds. Assume now that the assertion holds for at most $k - 1$ subjects and consider an instance J with k subjects. Let $\mathcal{M}^*$

be any maximum matching of J and matching $\mathcal{M}^G = \cup_{p=1}^k \mathcal{M}^p$ be the output of some realization of Greedy1 in subject order $1, 2, \ldots, k$. Observe that

$$|\mathcal{M}_1^*| \leq |\mathcal{M}_1^G|. \tag{1}$$

Take the assignment $\mathcal{M}^* \setminus \mathcal{M}_1^*$ and add individual matched pairs of $\mathcal{M}_1^G$ one by one in an arbitrary order. To be able to add the next pair of $\mathcal{M}_1^G$, say (a, s) for some $a \in A_{1,r}$, then we might need to displace at most one pair of $\mathcal{M}_{r,p}^*$ for some $r \neq 1$ to create free places (there is clearly no obstruction in subject 1, thanks to inequality (1)). So if $\mathcal{M}'$ is the set of displaced matched pairs outside $\mathcal{M}_1^*$ then $|\mathcal{M}'| \leq |\mathcal{M}_1^G|$. After the process we get a matching $\mathcal{M} = \mathcal{M}^* \cup \mathcal{M}_1^G \setminus (\mathcal{M}_1^* \cup \mathcal{M}')$ such that

$$|\mathcal{M}_1^G| = |\mathcal{M}_1| \geq \frac{1}{2}(|\mathcal{M}_1^*| + |\mathcal{M}'|) . \tag{2}$$

As $\mathcal{M}_1 = \mathcal{M}_1^G$, the matching $\mathcal{M} \setminus \mathcal{M}_1$ is a matching of the instance that has one less subject and that we got after the first phase of Greedy. Hence by the induction hypothesis we have that

$$\begin{aligned} |\mathcal{M}^G| - |\mathcal{M}_1^G| &= \\ &= |\mathcal{M}^G \setminus \mathcal{M}_1^G| \geq \frac{1}{2}|(\mathcal{M} \setminus \mathcal{M}_1)| = \\ &= \frac{1}{2}|(\mathcal{M}^* \setminus (\mathcal{M}_1^* \cup \mathcal{M}'))| = \frac{1}{2}(|\mathcal{M}^*| - (|\mathcal{M}_1^*| + |\mathcal{M}'|)) . \end{aligned} \tag{3}$$

Combination of inequalities (2) and (3) implies

$$|\mathcal{M}^G| = |\mathcal{M}_1^G| + |\mathcal{M}^G \setminus \mathcal{M}_1^G| \geq \frac{1}{2}(|\mathcal{M}_1^*| + |\mathcal{M}'|) + \frac{1}{2}(|\mathcal{M}^*| - (|\mathcal{M}_1^*)| + |\mathcal{M}\,'|)) = \frac{1}{2}|\mathcal{M}^*|,$$

hence the assertion follows. ■

applicant	type	acceptable schools	applicant	type	acceptable schools
b_1	$\{3,4\}$	s_1	a_1	$\{1,2\}$	s_1, s_2, s_3
b_2	$\{2,4\}$	s_2	a_2	$\{1,3\}$	s_1, s_2, s_3
b_3	$\{2,3\}$	s_3	a_3	$\{1,4\}$	s_1, s_2, s_3

Figure 4: Instance J for Example 2.

Example 2. Take the TAP instance with four subjects, six applicants $a_1, a_2, a_3, b_1, b_2, b_3$, three schools s_1, s_2, s_3 with all partial capacities equal 1 and the characteristics of applicants given in Figure 4. Here, the greatest size of a matching, namely 6, is achieved by assigning the pair of applicants a_i, b_i to school s_i for $i = 1, 2, 3$. Suppose that Greedy1 works with the order of subjects $1, 2, 3, 4$. There are 6 different maximum matchings for A_1. However, if the first phase of Greedy1 produces e.g. $\mathcal{M}_1^G = \{(a_1, s_2), (a_2, s_3), (a_3, s_1)\}$ then no further applicant can be matched and Greedy1 outputs a matching whose size is half of the maximum. So the bound of Theorem 2 is also tight, already for four subjects.

3.2 Algorithm Greedy2

Our second algorithm, called Greedy2, is given in Figure 5.

---

begin for $p := 1$ to k find a maximum cardinality matching $\mathcal{M}^p$ of applicants A_p;
keep $\mathcal{M}^j$ whose size is maximum;
add applicants from $A \setminus A_j$ arbitrarily to get a maximal matching
end

Figure 5: Algorithm Greedy2

Theorem 3 Greedy2 *is a* $\frac{k}{2}$*-approximation algorithm.*

Proof. Let $\mathcal{M}^*$ be a maximum matching in an instance of MAX-TAP. Then

$$|\mathcal{M}_1^*| + |\mathcal{M}_2^*| + \cdots + |\mathcal{M}_k^*| = 2|\mathcal{M}^*|$$

as each matched pair (a, s) is counted twice in the left-hand side, namely in $|\mathcal{M}_p^*|$ and $|\mathcal{M}_r^*|$ if $a \in A_{p,r}$. By Pigeonhole principle, at least one term on the left, say $|\mathcal{M}_1^*|$, has the size at least $|\mathcal{M}^*|/k$. As $|\mathcal{M}_1^*|$ is not greater than $\max\{|\mathcal{M}^p|, p = 1, 2, \ldots, k\}$, Algorithm Greedy2 outputs a matching of size at least $|\mathcal{M}^*|/k$. ■

Example 3. In the TAP instance of Figure 6, the maximum matching is

$$\mathcal{M} = \{(a_1, s_2), (a_2, s_2), (a_3, s_1)\}$$

of size 3. For each subject p, the cardinality of maximum matching $\mathcal{M}^p$ is 2, but if Greedy2 chooses $p = 1$ and matching $\mathcal{M}_1 = \{(a_1, s_1), (a_2, s_1)\}$ then the matching output by the algorithm will be of size 2. This is finally a tight example for Greedy2.

3.3 Algorithm Greedy3

Our last algorithm is based on a simple maximum cardinality bipartite matching algorithm, see Figure 7. First, we create the bipartite graph $G = (A, S, E)$ where the pair $\{a, s\} \in E$ if $s \in S(a)$, and $c_{p1(a)}(s) \geq 1$ and $c_{p2(a)}(s) \geq 1$. After a maximum cardinality

school	capacities for			applicant	type	acceptable schools
	1	2	3			
s_1	2	1	1	a_1	$\{1,2\}$	s_1, s_2
s_2	2	1	1	a_2	$\{1,3\}$	s_1, s_2
				a_3	$\{2,3\}$	s_1

Figure 6: Instance J for Example 3.

---

begin $\mathcal{M} := \emptyset$; $\ell = 1$;
 while $E \neq \emptyset$ **do**
 begin find a maximum cardinality matching $\mathcal{M}^\ell$ in G; $\mathcal{M} := \mathcal{M} \cup \mathcal{M}^\ell$;
  for all $(a, s) \in \mathcal{M}^\ell$ **do**
  begin $c_{p_1(a)}(s) := c_{p_1(a)}(s) - 1$; $c_{p_2(a)}(s) := c_{p_2(a)}(s) - 1$; delete vertex a **end**;
  delete all edges $\{b, s\}$ such that $c_p(s) = 0$ for at least one $p \in \mathbf{p}(b)$;
  $\ell := \ell + 1$
 end
end

Figure 7: Algorithm Greedy3

matching in G is found, partial capacities of schools are decreased accordingly, all edges corresponding to partial capacities that were made zero are erased and the process is repeated with remaining unassigned students.

Clearly, Greedy3 finds a maximal matching, so its approximation guarantee is 3. When applied to the instance in Examples 1, it gets a maximum matching. In Examples 2 and 3 it does not give better results (as far as the number of assigned applicants is considered) if an unfavourable maximum cardinality matching is chosen, but its time complexity is clearly much better: for one iteration only $O(mn\sqrt{n+m})$ operations are needed, if Hopcroft-Karp algorithm is used [9]. Finally, let us present an asymptotically tight example for the set of subjects $P = \{1, 2, 3\}$.

Example 4. Let

$$A = \{a_1, a_2, \ldots, a_{n-1}, b_0, b_1, \ldots, b_{n-1}, c_1, c_2, \ldots, c_{n-1}\},\ S = \{s_0, s_1, \ldots, s_{n-1}\}$$

and suppose that all schools are acceptable for each applicant. The types of applicants and capacities of schools are given in Figure 8.

First observe that all $3n-2$ applicants can be matched as follows: applicants $b_0, a_1, a_2, \ldots, a_{n-1}$ are assigned to s_0, while b_i and c_i to s_i, $i = 1, 2, \ldots, n-1$. However, if Greedy3 assigns in iteration 1 applicant b_0 to s_0 and applicants $a_1, a_2, \ldots, a_{n-1}$ to schools

$s_1, s_2, \ldots, s_{n-1}$ respectively, then the algorithm ends in this iteration and the number of assigned applicants is n. So, if n grows to infinity, the ratio of applicants assigned by Greedy3 to the maximum possible cardinality of a matching is 1/3.

schools	capacities for 1	2	3	applicants	type
s_0	1	n	$n-1$	$a_1, a_2, \ldots, a_{n-1}$	$\{2,3\}$
$s_1, s_2, \ldots, s_{n-1}$	2	1	1	$b_0, b_1, \ldots, b_{n-1}$	$\{1,2\}$
				$c_1, c_2, \ldots, c_{n-1}$	$\{1,3\}$

Figure 8: Instance J for Example 4.

4 Conclusion and open questions

We showed that the MAX-TAP problem, in spite of being intractable, allows relatively easy approximation algorithms. Notice, however, that the tight examples for all the proposed algorithms are such that the lower bound is achieved if in the first step an "incorrect" matching is chosen. Since there is always a choice leading to an optimal solution, a question is whether these algorithms could be refined to obtained a better approximation bound. In general, the obtained bounds leave a lot of space for improvement, even more so, as so far no lower bound of approximation has been obtained. Or, is it possible that MAX-TAP could be APX-complete? We are inclined to believe in this possibility because the NP-completeness provided in [2, 3] uses as the starting point 3-DIMENSIONAL MATCHING, that is APX-complete thanks to the result of Kann [8]. On the other hand, a result in [4] implies that the problem of *minimizing the number of unassigned students* does not admit a polynomial approximation algorithm with guarantee $n^{1-\varepsilon}$ for any $\varepsilon > 0$, if P$\neq$ NP.

Finally, it could be worthwile to study the MAX-TAP problem from the parameterized complexity angle. One possible parameter is the maximum partial capacity of schools.

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